HOMEWORK: Propositional Logic

1. Show that: $\vdash P\to P$ (i.e., construct an axiomatic proof of $P\to P$)

2. Show that $\vdash(\sim P\to P)\to P$

3. Show that $\sim\sim\sim P\vdash\sim P$ (i.e., construct an axiomatic proof from $\sim\sim\sim P$ to $\sim P$)

4. Consider the following (very strange) system of propositional logic. The definition of wffs is the same as for standard propositional logic, and the rules of inference are the same (just one rule: modus ponens); but the axioms are different:

Axioms: For any wffs ϕ and ψ, the following are axioms:

$\phi\to\phi$
$(\phi\to\psi)\to(\psi\to\phi)$

Show two facts about this system:

a) every theorem of this system has an even number of "~"s.
b) soundness is false for this system – i.e., some theorems are not valid formulas

5. Back to normal propositional logic. Show that:

a) $\vDash\sim(P\to Q)\to\sim Q$
b) $\vDash(P\to(Q\to R))\to(Q\to(\sim R\to\sim P))$
c) $\{P\to Q, Q\to R\}\vDash P\to\sim\sim R$

6. Given the definitions of the defined symbols $\vee$ and $\leftrightarrow$, show that for any valuation function V, and any wffs ψ and χ,

$V(\psi\vee\chi)=1$ iff either $V(\psi)=1$ or $V(\chi)=1$
$V(\psi\leftrightarrow\chi)=1$ iff $V(\psi)=V(\chi)$

7. Prove the following form of soundness: for any set of formulas, Γ, and any formula ϕ, if $\Gamma\vdash\phi$ then $\Gamma\vDash\phi$ (i.e., if ϕ is *provable from* Γ then ϕ is a semantic consequence of Γ.)