

## Homework: sequents, and variations and deviations from standard propositional logic

1. Prove the following sequents:

- a.  $P, Q, R \vdash P$
- b.  $P \rightarrow (Q \rightarrow R) \vdash (Q \& \sim R) \rightarrow \sim P$
- c.  $P \rightarrow Q, R \rightarrow Q \vdash (P \vee R) \rightarrow Q$

2. For each of the following truth functions, first find a sentence that symbolizes it in standard propositional logic (i.e., with  $\sim, \&, \vee, \leftrightarrow, \rightarrow$ ); then find a sentence that symbolizes it using just the sheffer stroke:

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|---|---|
| <ol style="list-style-type: none"> <li>a. <math>f(1,1)=1</math></li> <li><math>f(1,0)=0</math></li> <li><math>f(0,1)=0</math></li> <li><math>f(0,0)=1</math></li> </ol> | <ol style="list-style-type: none"> <li>b. <math>g(1,1,1)=1</math></li> <li><math>g(1,1,0)=0</math></li> <li><math>g(1,0,1)=1</math></li> <li><math>g(1,0,0)=1</math></li> <li><math>g(0,1,1)=1</math></li> <li><math>g(0,1,0)=1</math></li> <li><math>g(0,0,1)=0</math></li> <li><math>g(0,0,0)=1</math></li> </ol> |
|---|---|

3. Show that all truth functions can be defined using just  $\downarrow$  (nor). The truth table for  $\downarrow$  is the following:

$\downarrow$	1	0
1	0	0
0	0	1

4. Translate each of the following into polish notation:

- a.  $P \leftrightarrow \sim P$
- b.  $(P \rightarrow (Q \rightarrow (R \rightarrow \sim \sim (S \vee T))))$
- c.  $[(P \& \sim Q) \vee (\sim P \& Q)] \leftrightarrow \sim [(P \vee \sim Q) \& (\sim P \vee Q)]$

5. When  $P = 1, Q = 0, R = \#$ , calculate the truth values of the following formula, first using the Łukasiewicz tables, then the Kleene (strong) tables, then the Bochvar tables:

$$(\sim P \rightarrow R) \& (Q \vee R)$$

6. For each of the three tables (Łukasiewicz, Kleene, Bochvar), is the following a *valid* formula? In each case, justify your answer:

$$(P \& Q) \rightarrow (P \vee Q)$$

7. Extra credit: Can all truth functions can be defined using the following connective %? Justify your answer.

%	1	0
1	0	1
0	1	0