

MODAL PROPOSITIONAL LOGIC THEOREMS

K-Theorems

1. $\Box(P \& Q) \rightarrow (\Box P \& \Box Q)$
2. $\sim \Diamond(P \vee Q) \rightarrow (\sim \Diamond P \& \sim \Diamond Q)$
3. $\Diamond(P \& Q) \rightarrow (\Diamond P \& \Diamond Q)$
4. $\Box(\sim P \rightarrow P) \leftrightarrow \Box P$
5. $\Box(P \rightarrow \sim P) \leftrightarrow \sim \Diamond P$
6. $\Box P \rightarrow \Box(Q \rightarrow P)$
7. $\Box \sim P \rightarrow \Box(P \rightarrow Q)$
8. $\sim \Diamond(Q \& R) \leftrightarrow \Box(Q \rightarrow \sim R)$
9. $[\Box(P \rightarrow Q) \& \Box(P \rightarrow \sim Q)] \rightarrow \Box(P \rightarrow \sim P)$
10. $(\Box P \& \Box Q) \rightarrow \Box(P \& Q)$
11. $\Box(P \leftrightarrow Q) \rightarrow (\Box P \leftrightarrow \Box Q)$
12. $(\Box P \vee \Box Q) \rightarrow \Box(P \vee Q)$
13. $[\Box(Q \rightarrow P) \& \Box(\sim Q \rightarrow P)] \leftrightarrow \Box P$
14. $[\Box(P \rightarrow Q) \& \Box(P \rightarrow \sim Q)] \rightarrow \sim \Diamond P$
15. $\Box(P \vee Q) \rightarrow (\Box P \vee \Diamond Q)$
16. $[\Box(P \rightarrow Q) \& \Box(Q \rightarrow R)] \rightarrow \Box(P \rightarrow R)$
17. $(\Box P \& \Box Q) \rightarrow \Box(P \leftrightarrow Q)$
18. $\Diamond(P \vee Q) \leftrightarrow (\Diamond P \vee \Diamond Q)$
19. $(\Diamond P \& \Box Q) \rightarrow \Diamond(P \& Q)$
20. $[\Box(P \rightarrow Q) \& \Diamond(P \& R)] \rightarrow \Diamond(Q \& R)$
21. $\Diamond(P \rightarrow Q) \leftrightarrow (\Box P \rightarrow \Diamond Q)$
22. $\Diamond P \rightarrow (\Box Q \rightarrow \Diamond Q)$
23. $\Diamond[P \rightarrow (Q \& R)] \rightarrow [(\Box P \rightarrow \Diamond Q) \& (\Box P \rightarrow \Diamond R)]$
24. $[\Box \Diamond P \& \Diamond \Box(P \rightarrow Q)] \rightarrow \Diamond \Diamond Q$

D-Theorems

25. $\Box \Box P \rightarrow \Box \Diamond P$
26. $\Box \Box P \rightarrow \Diamond \Diamond P$
27. $[\Box P \& \Box(P \rightarrow Q)] \rightarrow \Diamond Q$
28. $\sim \Box(P \& \sim P)$

29. $\diamond(P \rightarrow P)$
30. $\sim(\Box P \& \Box \sim P)$
31. $\sim\Box(\Box P \& \Box \sim P)$
32. $\diamond\Box(P \& Q) \rightarrow \diamond\diamond P$
33. $(\diamond\sim P \vee \diamond\sim Q) \vee \diamond(P \& Q)$

T-Theorems

34. $\Box P \rightarrow \Box\diamond P$
35. $\diamond\Box P \rightarrow \diamond(P \vee Q)$
36. $[\Box P \& \diamond\Box(P \rightarrow Q)] \rightarrow \diamond Q$
37. $\diamond(P \rightarrow \Box Q) \rightarrow (\Box P \rightarrow \diamond Q)$

B-Theorems

38. $\diamond\Box P \rightarrow \Box\diamond P$
39. $\diamond\Box P \rightarrow \diamond\Box\diamond\Box P$
40. $\sim\diamond(\diamond\Box\diamond P \& \Box\sim P)$
41. $[\Box P \& \Box\diamond\Box(P \rightarrow Q)] \rightarrow \Box Q$

S4-Theorems

42. $(\diamond P \& \Box Q) \rightarrow \diamond(P \& \Box Q)$
43. $\Box P \rightarrow \Box\diamond\Box P$
44. $\Box\diamond P \rightarrow \Box\diamond\Box\diamond P$
45. $(\Box P \vee \Box Q) \rightarrow \Box(\Box P \vee \Box Q)$
46. $\Box[\Box(P \leftrightarrow Q) \rightarrow R] \rightarrow \Box[\Box(P \leftrightarrow Q) \rightarrow \Box R]$
47. $\Box\diamond\Box\diamond P \rightarrow \Box\diamond P$
48. $\diamond\Box P \rightarrow \diamond\Box\diamond\Box P$
49. $\diamond\Box\diamond\Box P \rightarrow \diamond\Box P$

S5-Theorems

50. $\diamond\diamond\diamond\Box P \leftrightarrow \Box P$
51. $\Box\diamond\diamond\Box P \leftrightarrow \Box\Box P$
52. $\Box(\sim P \vee \Box Q) \leftrightarrow (\Box\sim P \vee \Box Q)$
53. $\Box(\sim P \vee \diamond Q) \leftrightarrow (\sim\diamond P \vee \diamond Q)$
54. $(\Box P \vee \diamond Q) \rightarrow \Box(P \vee \diamond Q)$

$$55. \Box(P \vee \Diamond Q) \rightarrow (\Box P \vee \Diamond Q)$$

$$56. \Diamond(P \& \Diamond Q) \leftrightarrow (\Diamond P \& \Diamond Q)$$

$$57. (\Diamond P \& \Box Q) \leftrightarrow \Diamond(P \& \Box Q)$$

$$58. \Box(\Box P \rightarrow \Box Q) \vee \Box(\Box Q \rightarrow \Box P)$$

$$59. \Box[\Box(\Diamond P \rightarrow Q) \leftrightarrow \Box(P \rightarrow \Box Q)]$$