

Review Sheet: Exam 2, Modal Logic

PROBLEMS

1. **Symbolizations** in propositional modal logic, tense logic, and counterfactuals. Know the difference between the necessity of the consequence and the necessity of the consequent.
2. **Axiomatic proofs** in propositional modal logic, in systems K,D,T,B,S4,S5.

Strategies:

- a. **PL strategies:** you are allowed to write down any PL tautology at any time, and write down any PL consequence of earlier formulas at any time. In either case, the annotation is “PL”. NOTE: the modal logic strategies below always get you conditionals. So if you’re trying to prove something that isn’t a conditional, you will usually have to first prove a conditional, then use PL to change it into what you want.
 - b. **Proving a formula of the form $\Box\phi \rightarrow \Box\psi$,** where the conditional $\phi \rightarrow \psi$ is provable: first prove the conditional $\phi \rightarrow \psi$, then Necessitate, then distribute the \Box over the \rightarrow using $K\Box$.
 - c. **Proving a formula of the form $\Diamond\phi \rightarrow \Diamond\psi$,** where the conditional $\phi \rightarrow \psi$ is provable. As above, but use $K\Diamond$.
 - d. **Proving a formula of the form $\Box\phi_1 \rightarrow (\Box\phi_2 \rightarrow \dots (\Box\phi_n \rightarrow \Box\psi) \dots)$,** where the conditional $\phi_1 \rightarrow (\phi_2 \rightarrow \dots (\phi_n \rightarrow \psi) \dots)$ is provable: first prove the conditional, then necessitate, then use $K\Box$ (or K) repeatedly.
 - e. **Proving a formula of the form $\Box\phi_1 \rightarrow (\Box\phi_2 \rightarrow \dots (\Diamond\phi_n \rightarrow \Diamond\psi) \dots)$,** where the conditional $\phi_1 \rightarrow (\phi_2 \rightarrow \dots (\phi_n \rightarrow \psi) \dots)$ is provable. As above, but at the very end use $K\Diamond$ instead of $K\Box$. Note: for this to work, you must use $K\Diamond$ *after* all the uses of $K\Box$.
 - f. **Systems higher than K:** use the same strategies as above, but also look to use the characteristic axioms of the systems.
 - g. **Note on the strategies listed above:** each begins by proving a certain conditional. This conditional does *not* have to be a PL tautology. It *may* be a PL tautology, but all that’s required is that the conditional is provable in one way or another. The conditional might require modal logic to prove. It might, for instance, be the characteristic axiom of one of the higher systems, or it might be something provable using one of the strategies a-e.
3. **Countermodels** in propositional modal logic: coming up with models that show a given formula to be invalid in the various systems. I will give you a formula (as on the second homework assignment), and ask you which systems the formula is invalid in. You do this using the method of diagrams described in class. Begin by trying for a K-countermodel, then strengthen it as much as you can. In most cases, your answer will be a single countermodel, in the strongest system in which the formula is invalid. But in one case, you will need two countermodels: if the formula is valid in S5 but not valid in any weaker systems, you will need both a B and an S4-countermodel.
 4. **Validity proofs** in propositional modal logic. I may ask you to give a validity proof of a

formula in a specified system, or I may give you a formula and ask you which systems the formula is valid in. The easiest approach in the latter case is to first use the method of diagrams to find a countermodel in K, then attempt to strengthen the model into a countermodel in stronger systems. When you fail, the formula is probably valid in that system. In general your answer will be a single validity proof, in the weakest system in which the formula is valid. But in one case you will need two validity proofs: if the formula is invalid in T, but valid in both B and S4, then you will need both a B- and an S4-validity proof.

5. **Models for counterfactuals.** We only spent one day on this. So all I would ask you to do is come up with the picture of a model showing that a given formula is invalid, or show that a formula is valid by showing informally why there can be no countermodel (i.e., the attempt to construct the picture of a countermodel fails.)

CONCEPTS

6. **Basic concepts** of modal logic: the basic idea of possibility and necessity, different strengths of necessity, why truth tables won't work for modal logic (the \Box and \Diamond aren't truth-functional). Basic concepts of tense logic, including the tense operators, and possible worlds semantics of them. Basic concepts of counterfactuals: why counterfactuals differ from material conditionals and indicative conditionals, why counterfactuals are context dependent; the basic idea of the Lewis-Stalnaker semantics for counterfactuals.
7. **Grammar** of propositional modal logic: definition of a wff, official versus unofficial connectives.
8. **Provability** in propositional modal logic. Know the characteristic axioms of the systems, know their rules of inference, and know the definition of a theorem ("the last line of a proof, where a *proof* is defined as ...").
9. **Provability metatheorems:** substitution of equivalents (know what it is and how to use it; you don't need to be able to prove it), and the S5 reduction theorem (strings of modal operators reduce to the innermost one; you should know informally why this is true.)
10. **Semantics** of propositional modal logic: definition of frames, valuation function and models. Know the different formal properties of relations (symmetry, transitivity, reflexivity, seriality), and know which formal properties are required of the frames in the different systems.
11. **Semantic metatheorems:** know what soundness and completeness (for the various systems) are. Know roughly how to prove soundness. Know the significance of soundness: it means that if you can come up with a countermodel for a formula in a given system, then it is invalid, and so, given soundness, not provable in that system.
12. **The diagram of systems** (showing which systems are stronger than which). Understand its consequences for validity, invalidity, provability and unprovability.