Math Logic
Homework #1

Chapter 1

1. Where S is a finite set and T is an enumerable set, show that $S \cap T$ (the intersection of S and T; i.e., the set of things that are in both S and T) is enumerable.

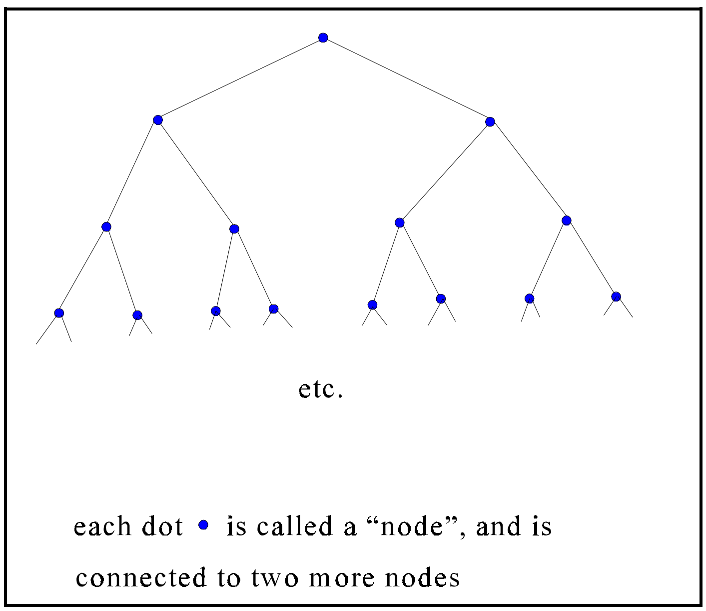
2. Show that the intersection of an enumerable set of enumerable sets is itself enumerable. (Where S is a set of sets, the intersection of S is defined as the set of all things that are members of each member of S.)

3. Let F be the set of all one to one functions that i) have a domain that's a subset of the positive integers, and ii) are onto a two element set {a,b}. Show that F is enumerable.

4. (Difficult). A *finite sequence of positive integers* is a finite, non-gappy list of positive integers. For example: <5,1000006,89,1263> is a 4-membered sequence. Show that the set of all finite sequences of positive integers is enumerable. (Hint: first prove that for all n, the set of all n-membered sequences of positive integers is enumerable, and then use this fact in your proof.)

Chapter 2

An infinite binary tree looks like this:



5. Show that the set of nodes of an infinite binary tree is enumerable.

6. Show that the set of infinite paths beginning at the origin down an infinite binary tree is *not* enumerable.

7. Where $\mathbb{N}^+$ is the set of positive integers, prove that the set of all one-to-one, total functions from $\mathbb{N}^+$ into $\mathbb{N}^+$ is not enumerable.

8. Prove that the set of all one-to-one, total functions from $\mathbb{N}^+$ *onto* $\mathbb{N}^+$ is not enumerable. (difficult)