Math Logic
Homework #2, Turing Machines

A. For each of the following tasks, if there is a Turing machine that performs the task, give a flow diagram of such a machine. If there is no such Turing machine, explain why.

1. The machine creates a "negative" of the entire tape to the right of the initial square -- i.e. switching ones to blanks and vice versa. More carefully: for any square to the right of the initial square (including the initial square), at some time that square gets reversed, and stays reversed forever after.

2. If the tape has a 1 anywhere, the machine halts scanning a 1.

3. As in 2, with the addition that if the tape is blank everywhere, the machine halts scanning a blank square.

B. Construct flow diagrams of Turing machines that compute the following functions from positive integers to positive integers, given the definition of "Turing machine M computes function f" stated in class.

4. $f(x,y) = x-y$ (Understand $x-y$ to be 1 if $y \geq x$)

5. $f(x,y)=x$

6. $f(x,y)$ = the average of x and y, rounded up to the nearest integer.

Extra credit

7. $f(x) = 2x$ in 7 states or less (6 is possible, apparently)

8. $f(x,y) = x-y$ in 14 states.

9. Prove that only what a Turing machine "sees" can affect its behavior. Hints: formulate it more rigorously, along the lines of "if a Turing machine executes a certain sequence of actions when started on a certain tape, then it will execute the same sequence of actions when started on any other tape that is like the first tape with respect to every square that the machine visited", but even more carefully. Then do some kind of induction.