Math Logic
Homework #3b, Chapter 7

1. Show that the following function is primitive recursive, by giving its name in official notation, with names for functions drawn solely from the initial primitive recursive functions: $f(x) = x^2 + 3x + 1$.

In all of the rest, you may assume that the functions listed on pp. 84-88 are primitive recursive. (I.e., you may use their names in official notation, and you may use symbols for them in unofficial notation. You may use unofficial notation unless otherwise specified.)

2. Show that the following function is primitive recursive, by giving it a name in official notation:

   f(0)=0; f(1)=1; f(2)=3; f(3)=6; f(4)=10; f(5)=15; etc.

   i.e., f(i)=f(i-1)+i

3. Say that a set of non-negative integers is primitive recursive iff its characteristic function is primitive recursive. (The characteristic function, f, of a set A of non-negative integers, is the function f such that f(n)=1 if $n \in A$; f(n)=0 if $n \notin A$.) Show that if A and B are primitive recursive sets, then so are $A \cap B$, $A \cup B$, and A-B. (note: A-B is the set of all things that are in A but not in B.)

4. Let Prime(x) = 1 if x is prime
   0 otherwise

   Show that Prime is primitive recursive. (Hint: general summation or general product may be helpful.)

5. Define function P as follows:

   P(0)=0
   P(n) = the n^{th} prime (where n>0)

   Show that P is primitive recursive. (Hint: it is a fact, and you may assume this fact, that for any prime, x, the next prime is less than or equal to x!+1.)

Extra Credit

6. Show that the function Fib(x) which lists the Fibonacci numbers (with an initial 0) is primitive recursive: Fib(0)=0 Fib(1)=1 where n>0, Fib(s(n)) = Fib(n-1) + Fib(n) (Hint: look at the trick on p. 93 for coding ordered pairs.)

7. Write minimization with primitive recursive bound (7.17, p. 87) in official notation. Note that the summation goes from 0 to some function $g(x_1...x_n)$.)