Tarski on Truth

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Phil Language

Alfred Tarski sought a "non-metaphysical" and mathematically precise definition of the term 'true'.

1. Criteria for an acceptable theory of truth

Material adequacy The theory must entail every sentence of the following form, where 'p' is replaced by any sentence of the language L, and 'X' is replaced by any name of that sentence:

(T) X is true-in-L if and only if p

Formal correctness The definition must conform to ordinary standards of mathematical rigor

Note:

- this defines truth for sentences, not for propositions (or ideas, or…)
- "To say of what is that it is, or of what is not that it is not, is true"
- Note the use-mention subtleties

2. The Liar

According to Tarski, because of the Liar Paradox, in some languages you simply can't give an adequate definition of truth.

(S) Sentence (S) is not true

- *Suppose (S) is true.* Then what (S) says is the case. But (S) says that (S) is not true. So (S) must not be true after all. So a contradiction results from the supposition that (S) is true.

- *Suppose (S) is not true.* But this is exactly what (S) says is the case. So (S) is true after all. So a contradiction results from the supposition that (S) is not true.

Contradiction either way!

According to Tarski, you get the liar paradox whenever a language is:

semantically closed: contains a truth predicate—i.e. a predicate obeying (T)—and also has the means to name all of its own sentences

classical: obeys the laws of ordinary logic

Tarski avoids the paradox by rejecting i). You can only introduce a truth predicate for an object language in a metalanguage.

3. Inductive definitions

base if x is a parent of y then x is an ancestor of y

induction if x is a parent of some ancestor of y, then x is an ancestor of y

nothing-else the only ancestors of y are things that can be shown to be ancestors of y using base and induction

4. A little language

Symbols of L:

Names: **Ted**, **Michael**

1-place predicates: **is a basketball player**, **is human**

2-place predicate: **admires**

Logical symbols: **∼**, **&**, ∨

Parentheses: **)**, **(**

Definition of formulas of L:

base if α is a name or variable and γ is a 1-place predicate then "$\alpha\pi$" is a formula (of L); if α and β are names or variables and γ is a two-place predicate then "$\alpha\gamma\beta$" is a formula

induction if ϕ and ψ are formulas then the following are all formulas: "**∼**ϕ", "(ϕ&ψ)", "(ϕ∨ψ)"

nothing else

5. Definition of truth

Definition of denotation:

- '**Ted**' denotes-in-L Ted Sider
- '**Michael**' denotes-in-L Michael Jordan
- nothing else denotes-in-L anything

Definition of application-in-L

- '**is a basketball player**' applies to an object, u, iff u is a basketball player
- '**is human**' applies to an object, u, iff u is human
- '**admires**' applies to an ordered pair, $\langle u, v\rangle$ iff u admires v

Definition of truth-in-L:

base If α is a name and γ is a one-place predicate then "$\alpha\gamma$" is true-in-L if and only if γ applies-in-L to the denotation-in-L of α. If α and β are names and γ is a two-place predicate then "$\alpha\gamma\beta$" is true-in-L iff γ applies-in-L to $\langle a, b\rangle$, where a and b are the denotations-in-L of α and β, respectively.

induction Where ϕ and ψ are formulas:

- "$\sim\phi$" is true-in-L if and only if ϕ is not true-in-L
- "$\phi \& \psi$ " is true-in-L if and only if ϕ is true-in-L and ψ is true-in-L
- "$\phi \vee \psi$ is true-in-L if and only if either ϕ is true-in-L or ψ is true-in-L