

MODAL PROPOSITIONAL LOGIC THEOREMS

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Useful Logic

K-Theorems

1. $\sim\Box\Diamond\Box P \leftrightarrow \Diamond\Box\Diamond\sim P$
2. $\sim\Diamond(P\vee Q) \rightarrow (\sim\Diamond P \wedge \sim\Diamond Q)$
3. $\Diamond(P\wedge Q) \rightarrow (\Diamond P \wedge \Diamond Q)$
4. $\Box(\sim P \rightarrow P) \leftrightarrow \Box P$
5. $\Box(P \rightarrow \sim P) \leftrightarrow \sim\Diamond P$
6. $\Box P \rightarrow \Box(Q \rightarrow P)$
7. $\Box\sim P \rightarrow \Box(P \rightarrow Q)$
8. $\sim\Diamond(Q\wedge R) \leftrightarrow \Box(Q \rightarrow \sim R)$
9. $[\Box(P \rightarrow Q) \wedge \Box(P \rightarrow \sim Q)] \rightarrow \Box(P \rightarrow \sim P)$
10. $(\Box P \wedge \Box Q) \rightarrow \Box(P \wedge Q)$
11. $\Box(P \leftrightarrow Q) \rightarrow (\Box P \leftrightarrow \Box Q)$
12. $(\Box P \vee \Box Q) \rightarrow \Box(P \vee Q)$
13. $[\Box(Q \rightarrow P) \wedge \Box(\sim Q \rightarrow P)] \leftrightarrow \Box P$
14. $[\Box(P \rightarrow Q) \wedge \Box(P \rightarrow \sim Q)] \rightarrow \sim\Diamond P$
15. $\Box(P \vee Q) \rightarrow (\Box P \vee \Diamond Q)$
16. $[\Box(P \rightarrow Q) \wedge \Box(Q \rightarrow R)] \rightarrow \Box(P \rightarrow R)$
17. $(\Box P \wedge \Box Q) \rightarrow \Box(P \leftrightarrow Q)$
18. $\Diamond(P \vee Q) \leftrightarrow (\Diamond P \vee \Diamond Q)$
19. $(\Diamond P \wedge \Box Q) \rightarrow \Diamond(P \wedge Q)$
20. $[\Box(P \rightarrow Q) \wedge \Diamond(P \wedge R)] \rightarrow \Diamond(Q \wedge R)$
21. $\Diamond(P \rightarrow Q) \leftrightarrow (\Box P \rightarrow \Diamond Q)$
22. $\Diamond P \rightarrow (\Box Q \rightarrow \Diamond Q)$
23. $\Diamond[P \rightarrow (Q \wedge R)] \rightarrow [(\Box P \rightarrow \Diamond Q) \wedge (\Box P \rightarrow \Diamond R)]$
24. $[\Box\Diamond P \wedge \Diamond\Box(P \rightarrow Q)] \rightarrow \Diamond\Diamond Q$

D-Theorems

25. $\Box\Box P \rightarrow \Box\Diamond P$
26. $\Box\Box P \rightarrow \Diamond\Diamond P$
27. $\Box P \rightarrow \Diamond(Q \rightarrow P)$
28. $\sim\Box(P \wedge \sim P)$

29. $[\Box P \wedge \Box (P \rightarrow Q)] \rightarrow \Diamond Q$
 30. $\sim(\Box P \wedge \Box \sim P)$
 31. $\Diamond\{[(P \rightarrow Q) \rightarrow P] \rightarrow P\}$
 32. $\sim\Box[\Box(P \wedge Q) \wedge \Box(P \rightarrow \sim Q)]$
 33. $(\Diamond \sim P \vee \Diamond \sim Q) \vee \Diamond(P \wedge Q)$

T-Theorems

34. $\Box P \rightarrow \Box \Diamond P$
 35. $\Diamond \Box P \rightarrow \Diamond(P \vee Q)$
 36. $[\Box P \wedge \Diamond \Box (P \rightarrow Q)] \rightarrow \Diamond Q$
 37. $\Diamond(P \rightarrow \Box Q) \rightarrow (\Box P \rightarrow \Diamond Q)$

B-Theorems

38. $\Diamond \Box P \rightarrow \Box \Diamond P$
 39. $\Diamond \Box P \leftrightarrow \Diamond \Box \Diamond \Box P$
 40. $\sim \Diamond(\Diamond \Box \Diamond P \wedge \Box \sim P)$
 41. $[\Box P \wedge \Box \Diamond \Box (P \rightarrow Q)] \rightarrow \Box Q$

S4-Theorems (43, 44, 47, 48, and 49 are also B-theorems)

42. $(\Diamond P \wedge \Box Q) \rightarrow \Diamond(P \wedge \Box Q)$
 43. $\Box P \rightarrow \Box \Diamond \Box P$
 44. $\Box \Diamond P \rightarrow \Box \Diamond \Box \Diamond P$
 45. $(\Box P \vee \Box Q) \rightarrow \Box(\Box P \vee \Box Q)$
 46. $\Box[\Box(P \leftrightarrow Q) \rightarrow R] \rightarrow \Box[\Box(P \leftrightarrow Q) \rightarrow \Box R]$
 47. $\Box \Diamond \Box \Diamond P \rightarrow \Box \Diamond P$
 48. $\Diamond \Box P \rightarrow \Diamond \Box \Diamond \Box P$
 49. $\Diamond \Box \Diamond \Box P \rightarrow \Diamond \Box P$

S5-Theorems

50. $\Diamond \Diamond \Diamond \Box P \leftrightarrow \Box P$
 51. $\Box \Diamond \Diamond \Box P \leftrightarrow \Box \Box P$
 52. $\Box(\sim P \vee \Box Q) \leftrightarrow (\Box \sim P \vee \Box Q)$

$$53. \quad \Box(\sim P \vee \Diamond Q) \leftrightarrow (\sim \Diamond P \vee \Diamond Q)$$

$$54. \quad (\Box P \vee \Diamond Q) \leftrightarrow \Box(P \vee \Diamond Q)$$

$$55. \quad \Diamond(P \wedge \Diamond Q) \leftrightarrow (\Diamond P \wedge \Diamond Q)$$

$$56. \quad (\Diamond P \wedge \Box Q) \leftrightarrow \Diamond(P \wedge \Box Q)$$

$$57. \quad \Box(\Box P \rightarrow \Box Q) \vee \Box(\Box Q \rightarrow \Box P)$$

$$58. \quad \Box[\Box(\Diamond P \rightarrow Q) \leftrightarrow \Box(P \rightarrow \Box Q)]$$