

## Mixed Modal Propositional Logic Wffs

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For practice, you might go through each wff and give a falsifying model for every system in which the wff is not valid, and a semantic validity proof for every system in which the wff is valid.

1.  $\Box[P \rightarrow \Diamond(Q \rightarrow R)] \rightarrow \Diamond[Q \rightarrow (\Box P \rightarrow \Diamond R)]$
2.  $\Box(P \vee \Diamond Q) \rightarrow (\Box P \vee \Diamond Q)$
3.  $\Diamond(P \wedge \Diamond Q) \rightarrow (\Box \Diamond P \rightarrow \Diamond \Box P)$
4.  $\Box(\Diamond P \leftrightarrow \Diamond Q) \rightarrow \Box(P \leftrightarrow \Box Q)$
5.  $\Diamond(\Diamond P \wedge \sim Q) \vee \Box(P \rightarrow \Box Q)$
6.  $[\Box P \wedge \Box(\sim P \vee Q)] \rightarrow \Diamond Q$
7.  $\Diamond \Box \Box P \rightarrow \Diamond \Box P$
8.  $\Diamond \Box P \rightarrow \Diamond(Q \rightarrow P)$
9.  $(\Box P \wedge \Box Q) \rightarrow \Box(P \leftrightarrow Q)$
10.  $\Diamond \Box P \rightarrow \Diamond \Box \Diamond \Box P$
11.  $[\Diamond(P \wedge Q) \vee \Diamond(P \wedge R)] \rightarrow \Diamond P$
12.  $\Diamond(P \vee \sim P) \rightarrow (P \wedge \sim P)$
13.  $\Box(P \leftrightarrow Q) \rightarrow (\Box P \leftrightarrow \Box Q)$
14.  $\Box(P \leftrightarrow Q) \rightarrow \Box(\Box P \leftrightarrow \Box Q)$
15.  $\Box(P \wedge Q) \rightarrow \Box \Box(\Diamond P \rightarrow \Diamond Q)$
16.  $\Box \Diamond P \rightarrow \Diamond \Box P$
17.  $\Diamond(\Diamond P \wedge \sim Q) \vee \Box(P \rightarrow \Box Q)$
18.  $\Diamond \Box P \leftrightarrow \Box \Diamond P$

$$19. \quad \Box(\Box P \rightarrow Q) \vee \Box(\Box Q \rightarrow P)$$

$$20. \quad \Box(\Box P \rightarrow Q) \rightarrow \Box(\Box P \rightarrow \Box Q)$$

$$21. \quad \Diamond \Diamond \Box P \leftrightarrow \Box P$$

$$22. \quad \Diamond \Diamond P \rightarrow \Box \Diamond P$$

$$23. \quad \Box[P \rightarrow \Box(Q \rightarrow R)] \rightarrow [Q \rightarrow \Box(P \rightarrow R)]$$

$$24. \quad \Box[\Box(P \leftrightarrow Q) \rightarrow \Diamond Q] \rightarrow \Box[\Box(P \leftrightarrow Q) \rightarrow Q]$$

$$25. \quad \Box[\Box(P \rightarrow \Box P) \rightarrow \Box P] \rightarrow (\Diamond \Box P \rightarrow \Box P)$$