Quantified Modal Logic Wffs

Ted Sider
Useful Logic

Directions Assuming the constant domains definition of a model, for each formula, give a validity proof if the wff is valid, and a countermodel if it is invalid. Indicate whether and how the status of the wff would change given the varying domains definition of a model.

1. $\forall x \forall y(x \neq y \rightarrow \Box x \neq y)$
2. $\forall x \Box \exists y\, x{=}y$
3. $\exists x \Box x{=}a$
4. $\forall x \Box \forall y\, x{=}y$
5. $\Box \exists x Fx \rightarrow \Diamond \forall x Fx$
6. $\Diamond \forall x Fx \rightarrow \exists x \Diamond Fx$
7. $\Diamond \forall x Fx \rightarrow \sim \exists x \Box \sim Fx$
8. $(\Diamond Fa \wedge \Diamond Ga) \rightarrow \Diamond (Fa \wedge Ga)$
9. $\exists x \Diamond Rax \rightarrow \Diamond \Box \exists x \exists y Rxy$
10. $\Box \forall x(Fx \rightarrow Gx) \rightarrow (\forall x \Box Fx \rightarrow \Box \forall x Gx)$
11. $\Box \forall x(Fx \vee Gx) \rightarrow \forall x(\Box Fx \vee \Box Gx)$
12. $\exists x \Box (Fx \vee Gx) \rightarrow \Box (\forall x Fx \vee \exists x Gx)$
13. $\forall x(Fx \rightarrow \Diamond Gx) \rightarrow \Diamond \forall x(Fx \rightarrow Gx)$
14. $\forall x(\Box Fx \vee \Box Gx) \rightarrow \Box \forall x(Fx \vee Gx)$
15. $\Box \forall x(Fx \rightarrow Gx) \rightarrow \forall x(Fx \rightarrow \Box Gx)$
16. $(\Box \forall x(Fx \rightarrow \Box Fx) \wedge \Diamond \exists x Fx) \rightarrow \Box \exists x Fx$
17. $\exists x(Nx \wedge \forall y(Ny \rightarrow y{=}x) \wedge \Box Ox) \rightarrow \Box \exists x(Nx \wedge \forall y(Ny \rightarrow y{=}x) \wedge Ox)$