# Field's Nominalism

Ted Sider
Philosophy of Mathematics

Field takes mathematical discourse at face value. But he is a *nominalist*—he thinks there are no abstract entities. Thus mathematical sentences like "there is an even number greater than 2" are false.

Field's response to the indispensability argument is that mathematics is not indispensable since its usefulness does not require its truth.

## 1. Conservativity

Field's central idea is that mathematics is useful because it *simplifies reasoning*. And it can be useful in this way despite being false because it is *conservative*:

**Mathematics is conservative** Let $N_1, \ldots$ and $C$ be any nominalist statements, and let $T$ be any mathematical theory. Then if $C$ is a consequence of $N_1, \ldots$ together with $T$, $C$ must also be a consequence of $N_1, \ldots$ alone.

That is: adding mathematical "side premises" to nominalistic premises doesn't give you any new nominalistic conclusions. Thus it's legitimate for a nominalist to use platonistic "side premises" in deriving nominalistic conclusions from nominalistic premises.

But why not just directly produce the direct, purely nominalist, argument? Because the platonist argument is often much simpler.

## 2. Illustration of conservativity: arithmetic

($N_1$) There are exactly twenty-one aardvarks.

($N_2$) On each aardvark there are exactly three bugs

($N_3$) Each bug is on exactly one aardvark

Therefore:

($C$) There are exactly sixty-three bugs.

The numerical expressions are intended in the adjectival sense. Thus ($N_2$) (e.g.) is really:

$$\forall x(Ax \to \exists y \exists z \exists w(By \wedge Bz \wedge Bw \wedge Oyx \wedge Ozx \wedge Owx \wedge \forall v((Bv \wedge Ovx) \to (v = y \vee v = z \vee v = w))))$$

($C$) follows from ($N_1$)–($N_3$). This is hard to demonstrate nominalistically, but much easier using standard (impure) set theory and arithmetic:

1. Given ($N_1$) and set theory, the number of elements of the set of aardvarks is 21.
2. Given ($N_2$) and set theory, there is a function mapping each aardvark to the set of bugs on that aardvark, the number of elements of which is 3.
3. Given ($N_3$) and set theory, these sets of bugs form a partition of the set of all bugs.
4. Given 1–3 and set theory, the number of elements of the set of all aardvarks is $21 \times 3$.
5. Given arithmetic, $21 \times 3 = 63$.
6. Given 4, 5, and set theory, ($C$) is true.

## 3. Why think that mathematics is conservative?

*Pure* mathematical theories are conservative simply because they contain only mathematical vocabulary. But for that same reason, pure *physical* theories are also conservative with respect to, e.g., macroscopic vocabulary.

However, *impure* physical theories are not conservative. (An impure theory of electrons will say something about how facts about electrons relate to macro-facts, such as that streams of electrons are produced by batteries, can pass through wires, and produce heat when they do. Adding this to purely macroscopic premises, such as that you've connected the terminal of a battery with a wire, will imply new consequences that the macroscopic premises didn't imply on their own: that the wire will heat up.) But even impure mathematical theories, such as impure set theory, are presumably conservative.

## 4. Nominalizing physics

The arguments that Conservativity concrns must have premises and conclusions that are nominalist. But physics, it seems like the premises and conclusions are themselves about mathematics.

The main part of Field's view is an attempt to show that we can *restate* physics in such a way that it doesn't make reference to mathematical entities at all.

For instance, instead of speaking of mass using numbers ("the mass of this object is 30 grams"), we can instead say things like this:

- Physical object $a$ is *as-or-more-massive-than* physical object $b$

  "$a \succeq b$"

- Physical object $a$ "mass-concatenates" physical objects $b$ and $c$

  "$Cabc$"

Instead of saying that object $a$ is 15 g in mass and object $b$ is 30 g in mass, Field would say: $Cbaa$.

We can then use impure mathematics to prove that there are functions that *represent* nominalistic statements, such as statements about $\succeq$ and $C$.

**mass function:** a function $f$ from objects to real numbers such that:

$$f(x) \geq f(y) \text{ iff } x \succeq y$$
$$f(x) + f(y) = f(z) \text{ iff } Cxyz$$

For instance, if $Cbaa$, then for some mass function $m$, $m(a) = 15$ and $m(b) = 30$.

Field's overall idea: 1. We "nominalize" physical theories. 2. We use mathematics to prove the existence of mass functions, pressure functions, temperature functions, and the like. 3. We use these functions to simplify the derivation of nominalistic conclusions from nominalistic premises. 4. Since the mathematics used is conservative, the conclusions we thus derive will always follow solely from the nominalistic premises alone, and thus do not depend on the mathematics being true.

## 5. Infinitely many concrete objects

Field's method for "nominalizing" physical theories assumes the existence of infinitely many points of physical space. Field denies that this compromises his nominalism:

> The nominalistic objection to using real numbers was not on the grounds of their [cardinality] or of the structural assumptions (e.g., Cauchy completeness) typically made about them. Rather, the objection was to their abstractness: even postulating *one* real number would have been a violation of nominalism … Conversely, postulating uncountably many *physical* entities … is not an objection to nominalism; nor does it become any more objectionable when one postulates that these physical entities obey structural assumptions analogous to the ones that platonists postulate for the real numbers. (Field, quoted in Shapiro pp. 232–3)

## 6. Metalogical concerns

When Field speaks of statements "following" or being "consequences" of one another (as in the statement of conservativity), what does he mean? Both model-theoretic and proof theoretic accounts seem to presuppose abstract entities. Field's reply: primitivism about logical consequence

## 7. How many physical theories can be nominalized?