Logicism: Neo-Fregeanism

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1. Neo-Fregeanism

Hume's Principle the number of the Fs = the number of the Gs if and only if F and G are equinumerous.

Frege's Basic Law V The extension of F = the extension of G if and only if: for any object x, x has F if and only if x has G

We now know that arithmetic can be derived solely from Hume's Principle (in second-order logic), without the need for Basic Law V, and that Hume's Principle—unlike Basic Law V—is (probably) not inconsistent. This has recently led some to adopt the following view:

Neo-Fregean view of arithmetic Hume's Principle is a definition of 'number of'. Since arithmetic logically follows from this definition, logicism for arithmetic is vindicated.

2. Caesar problem

Frege thought that Hume's Principle couldn't be a definition because it doesn't settle questions like whether the number of fish in the ocean = Julius Caesar.

NeoFregeans might say that the answers to such questions are *vague* (or "unsettled"). This would fit with the abstract/structural conception of mathematics, since those questions are about the nonstructural features of numbers.

3. Implying objects

Objection: Hume's Principle can't be a definition because it implies the existence of infinitely many objects. Implying the existence of even one object disqualifies something from being a definition, according to Kant.

4. Implicit definition

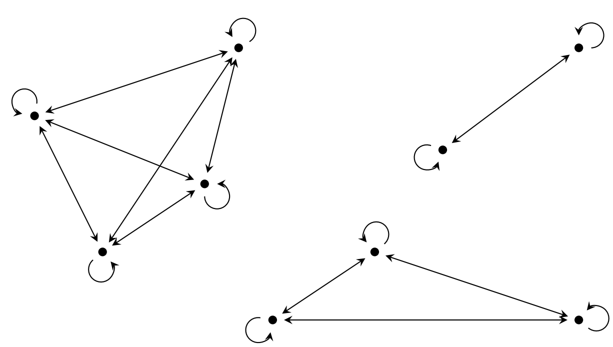
Objection: Hume's Principle isn't an "explicit" definition, since it doesn't give a synonym for 'number of'. Rather, it attempts to *select* that phrase's meaning:

> Consider all of the relations between concepts and objects in the entire world. You want to know which one I'm talking about, when I say 'number of'? I'll tell you: it's the one that satisfies Hume's Principle. That is, it is the relation, R, such that Hume's Principle comes out true if 'number of' is interpreted as meaning R.

But that means that in order to know that Hume's Principle is true, you'd need to know that some such relation R exists, which seems to require already knowing that infinitely many objects exist.

5. Definition by abstraction

Neo-Fregean reply: Hume's Principle is knowable a priori because it is a definition by *abstraction*, in which we begin with an equivalence relation (relation that is reflexive, symmetric, and transitive), and introduce objects corresponding to the relation's equivalence classes:



Example: begin with the equivalence relation *(full) sibling of*, and introduce "sib-squads" by abstraction:

> The sib-squad of x = the sib-squad of y if and only if x is a sibling of y

Example: begin with the equivalence relation *is parallel to* over lines, and introduce "directions" by abstraction:

> The direction of line $l_1 =$ the direction of line l_2 if and only if l_1 is parallel to l_2

Neo-Fregeans' attitude toward objects introduced by abstraction is similar to "ontological deflationism", according to which ontological questions such as whether there exist people, tables, molecules, planets, as opposed to there merely existing subatomic particles in various configurations, are "trivial", and not "substantive".

6. Bad company objection

Objection: definitions by abstraction aren't always legitimate, because sometimes they are inconsistent.

Example: begin with the equivalence relation *having exactly the same instances* (i.e., the relation holding between F and G iff for any object x, x has F iff x has G). Introduce new objects, "extensions", by abstraction:

> The extension of $F =$ the extension of G if and only if: for any object x, x has F if and only if x has G

This is just Basic Law V, which, as Russell showed, is false.

Reply: consistent abstraction principles are legitimate.

Counter-reply: to know that an abstraction principle is legitimate, we would need to prove that it is consistent, which would require already knowing some mathematics.

Counter-reply: we can't accept all consistent abstraction principles, since some individually consistent abstraction principles are inconsistent with each other. Boolos defined this equivalence relation: F and G *differ evenly* if and only if the number of objects falling under F but not G, or under G but not F, is even (and finite). Then introduce "parities" by abstraction:

Parity Principle The parity of $F =$ the parity of G if and only if F and G differ evenly

The Parity Principle is consistent, as is Hume's Principle, but they are inconsistent with each other (Hume's Principle implies that there are infinitely many things, and the Parity Principle implies that there are only finitely many things).

7. Can the account be extended?

Can analysis, set theory, and other branches of mathematics be based on definitions by abstraction?