Web of Belief

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Philosophy of Mathematics

1. The web of belief

1.1 Plato + Mill = Quine

Main problem for Plato: how do we know?

Main problem for Mill: most mathematical statements (especially in abstract mathematics) don't have direct empirical correlates.

Quine accepts a blend of platonism and empiricism. Quine is a realist about mathematics and mathematical entities, but he takes our knowledge of mathematics to derive from experience (without positing Gödelian intuition).

1.2 Holism

Simplistic picture of empirical justification: for each belief there is particular kind of observation that would conclusively verify it. Problem: nothing conclusively verifies that the sun always rises in the morning.

Less simplistic picture: for each belief there is particular kind of observation that would i) refute the belief if it's wrong, and ii) add to its confirmation if it's right. Problem: doesn't fit statements of physics about unobservable entities.

Better picture: theoretical beliefs make many predictions, and the more those predictions come true, the more the beliefs are justified.

Quine adds holism: an experiment confirms one's entire "web of belief", not just a particular isolated bit, since the entire web is needed to derive predictions.

> The totality of our so-called knowledge or beliefs, from the most casual matters of geography and history to the profoundest laws of atomic physics or even of pure mathematics and logic, is a man-made fabric which impinges on experience only along the edges. Or, to change the figure, total science is like a field of force whose boundary conditions are experience. A conflict with experience at the periphery occasions

> readjustments in the interior of the field. But the total field is so underdetermined by its boundary conditions, experience, that there is much latitude of choice as to what statements to reevaluate in the light of any single contrary experience. No particular experiences are linked with any particular statements in the interior of the field, except indirectly through considerations of equilibrium affecting the field as a whole. (Quine, 1951, pp. 42–3)

1.3 Mathematics supported by observation

Quine's epistemology of mathematics: a web of belief with a good track record—a history of making predictions that came out true—is supported as a whole. This includes the "interior" parts, not only those about unobservable physical objects, but also those about mathematical objects. For all these parts played a role in generating the track record.

1.4 Indispensability argument

But might there be a better web of beliefs with a good track record but containing nothing about mathematics?

Not a *good*—i.e., explanatory—web of belief, according to Quine and Putnam. For a good web must have laws of physics, and thus must have mathematics.

1.5 A realist approach to applied mathematics

But this re-raises the question about how facts about abstract mathematical objects can be applied to the physical world. Here is one answer.

Pure mathematical objects don't involve any nonmathematical objects. Examples: the natural number 77, the real number π, the empty set $\varnothing$, and the function $f(x) = x^2$ (the function that maps each real number to its square).

Impure mathematical objects involve some nonmathematical objects. Example: sets containing nonmathematical objects (e.g., $\{75, \pi, \text{Mars}\}$ or $\{\text{Allen Iverson}\}$).

Another (important) example: functions that map nonmathematical objects to mathematical objects, such as the following functions m and DP:

$$\begin{aligned} m(x) &= \text{ the mass of physical object } x \text{ in kilograms} \\ DP(x) &= \text{ the number of dollars in the pockets of person } x \end{aligned}$$

Thus we can represent nonmathematical properties of particular objects:

$DP(\text{Ted}) = 5$ (I have five dollars in my pocket)

$m(\text{Ted}) = 77$ (my mass is 77 kilograms)

And we can also state general laws:

$$\text{For any ideal gas, } x,\ P(x)V(x) = n(x)RT(x)$$

where P, V, T, and n are the following functions from physical objects to real numbers:

$$\begin{aligned} P(x) &= \text{ the pressure of } x \text{ in pascals} \\ V(x) &= \text{ the volume of } x \text{ in cubic meters} \\ T(x) &= \text{ the temperature of } x \text{ in degrees Kelvin} \\ n(x) &= \text{ the number of moles of } x \end{aligned}$$

and R is a certain real number (approximately 8.314).

Illustration: Suppose we learn by observation that for a certain ideal gas, g:

$$\begin{aligned} P(g) &= 100000 \\ V(g) &= 17 \\ n(g) &= 700 \end{aligned}$$

Suppose we also know that the ideal gas law is true:

$$\text{For any ideal gas, } x,\ P(x)V(x) = n(x)RT(x)$$

We can then infer from the ideal gas law, by pure logic, that:

$$P(g)V(g) = n(g)RT(g)$$

We can then substitute in the values of $P(g)$, $V(g)$, and $n(g)$, and use pure mathematics to conclude that:

$$T(g) = \frac{P(g)V(g)}{n(g)R} = \frac{100000 \cdot 17}{700 \cdot 8.314} = 292$$

That is: the temperature of the gas g is 292° K (about 66° F).

Concern: how can we learn facts like $V(g) = 17$ by observation?

Answer: Our web of belief contains connections between impure functions and observable matters, such as:

- If one object, x, fits inside another object, y, then $V(x) \leq V(y)$.
- If a solid object, x, is not water soluble, and is submerged in an initially full container of water, and a quantity y of water spills out, then $V(x) = V(y)$.
- If some object z is made up of two nonoverlapping objects x and y, then $V(z) = V(x) + V(y)$.

1.6 Deductivism and application

How will a deductivist understand impure mathematical statements, such as 'm(Ted) = 77'?

It doesn't mean: ' 'm(Ted) = 77' is provable from the axioms of pure mathematics'. For those axioms say nothing about the particular function m.

Perhaps it means ' 'm(Ted) = 77' is provable from the axioms of pure mathematics plus M', where M is some added axioms about m? But M will need to be the *right* axioms, whereas deductivists want to say that the axioms are arbitrarily chosen.

References

Quine, W. V. O. (1951). "Two Dogmas of Empiricism." *Philosophical Review* 60: 20–43. Reprinted in Quine 1953: 20–46.

— (1953). *From a Logical Point of View*. Cambridge, MA: Harvard University Press.