WILLIAMSON'S *Modal Logic as Metaphysics*

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Modality seminar

1. Methodology

> The title of this book may sound to some readers like Good as Evil, or perhaps Cabbages as Kings. If logic and metaphysics appear disjoint, the reason is not just the lingering spell of a logical positivist conception of metaphysics as cognitively meaningless and logic as cognitively meaningful but analytic. Many contemporary philosophers who acknowledge metaphysics as continuous with the rest of science are still inclined to assign logic a more special status. They see it as a neutral referee of disputes between scientific theories, including metaphysical theories, blowing the whistle when the rules are broken, not as a disputing party in its own right. If so, logic says nothing over which there could be such a dispute, on pain of non-neutrality; thus logical theories are quite different in status from scientific theories. This book is written in the contrary conviction that, just as metaphysics is much more like the rest of science than was once thought, so too is logic. Indeed, one role for logic is to supply a central structural core to scientific theories, including metaphysical theories, in essence no more above dispute than any other part of those theories. (Williamson, 2013, p. x)

> The study of modal logic takes many legitimate forms…In this book it has taken the form of a metaphysical enquiry. We fixed interpretations of the modal operators, as expressing metaphysical possibility and necessity, and of the quantifiers, as unrestricted, in accord with the ambitions of metaphysics. Modal logic in this form aims to discover which generalizations in such terms are true. The true generalizations constitute a quantified modal logic, but we do not know ahead of enquiry which one. At least in this area of philosophical logic, our task is not to justify principles that already play a fundamental role in our thinking. Rather, it is in a scientific spirit to build and test theories that codify putatively true generalizations of the sort at issue, to find out which are true…
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> In some looser ways, the methodology of this book is akin to that of a natural science. Both are abductive. Very general theories are formulated in a formal notation that facilitates complex rigorous deductions of their consequences. The theories are judged partly on their strength, simplicity, and elegance, partly on the fit between their consequences and what is

independently known. The fit has at least two dimensions. Theories should not entail anything we are in a position to falsify, since then they are false. Equally, the more they entail of what we are in a position to verify independently, the better. 'Entail' here means by the standards of the theory in question, rather than by the correct standards, since we are trying to find out what the latter are: logic here is no mere background framework but the very thing at issue. (Williamson, 2013, pp. 423–4)

...logic is concerned with the real world just as truly as zoology, though with its more abstract and general features. (Russell, 1919, p. 169)

Multiplying entities is sometimes a necessity for the sake of theoretical plausibility, because the alternative is massive loss of simplicity, elegance, and economy in principles. (Williamson, 2013, p. 9)

2. Necessitism stated

Necessitism $\forall x \Box \exists y\, y = x$ ("Everything necessarily is something")

Barcan Schema $\Diamond \exists x A \rightarrow \exists x \Diamond A$

Williamson accepts an ontology of necessarily existing but contingently "concrete" entities.

3. A simple argument

1. $\vdash t = t$ (reflexivity of identity)
2. $\vdash \exists y\, y = t$ (1, existential generalization)
3. $\vdash \Box \exists y\, y = t$ (2, necessitation)

Existential generalization $A(t) \vdash \exists x\, A(x)$

Free existential generalization $A(t) \vdash \exists x\, x = t \rightarrow \exists x\, A(x)$

4. Prior's system Q

Main idea: if a proposition is "about" an individual, then that proposition would not have existed, and hence would not have been true, had the individual not existed.

Thus even though $Ht \to Ht$ is a logical truth, $\Box(Ht \to Ht)$ is false. So Prior rejects the rule of necessitation.

Though $Ht \to Ht$ isn't necessary, it's "weakly necessary": $\sim\Diamond\sim(Ht \to Ht)$.

Williamson's main objection: you can't say "Ted might have failed to exist" in Q:

- $\Diamond\sim\exists x\, x = t$ isn't true
- Although $\sim\Box\exists x\, x = t$ is true, it doesn't convey the intended content since *any* formula of the form $\sim\Box A(t)$ is true

5. The being constraint

Being constraint: ("Having properties requires existence")

$$\Box\forall x\Box(Fx \to \exists z\, x{=}z) \qquad \text{(monadic case)}$$

$$\Box\forall x\Box\forall y\Box(Rxy \to (\exists z\, x{=}z \wedge \exists z\, y{=}z)) \qquad \text{(dyadic case)}$$

$$\Box\forall x_1\Box\ldots\Box\forall x_n\Box(Rx_1\ldots x_n \to (\exists z\, x_1{=}z \wedge \ldots \wedge \exists z\, x_n{=}z)) \qquad \text{(general case)}$$

> How could a thing be propertied were there no such thing to be propertied? How could one thing be related to another were there no such things to be related? (Williamson, 2013, p. 148)

Model-theoretically, the being constraint corresponds to the domain constraint: predicate extensions at a world must be drawn from that world's domain.

6. The being constraint and the simple argument

The contingentist could say: $t = t$ is a logical truth, but by the being constraint, $\exists x\, x = t$ isn't true at a world in which t doesn't exist, so $\Box\exists x\, x = t$ isn't true. (So the rule of necessitation is rejected.)

7. Serious actualism

8. A contingentist argument against the being constraint

"Logic itself ensures the self-identity of x; so:

(6) $\Box\forall x\Box x = x$

Now suppose for reductio that (5) is true:

(5) $\Box\forall x\Box(x = x \rightarrow \exists z\, x = z)$

(5) and (6) imply necessitism:

Necessitism $\Box\forall x\Box\exists z\, x = z$

So I must reject (5). But (5) is an instance of the being constraint."

Contingentists who accept (5) should, TW says, accept the being constraint in general. Self-identity is the easiest property to have; so if you can't even have that without existing, you surely can't have any properties at all without existing.

9. Being constraint and λ-abstraction

1. $\Diamond\sim\exists z\, t = z$ (contingentists accept this)
2. $\Box(\sim\exists z\, t = z \rightarrow \exists z\, t = z)$ (being constraint?)
3. Therefore, $\Diamond(\sim\exists z\, t = z \wedge \exists z\, t = z)$

Problem for contingentists who accept the being constraint? No: 2 isn't an instance of the being constraint. Distinguish:

$\Box\forall x\Box(Fx \rightarrow \exists z\, x{=}z)$ (monadic being constraint)

$\Box\forall x\Box(A \rightarrow \exists z\, x{=}z)$ (something else)

Objection: what is the deep distinction between the following?

"$\sim\exists z\, a = z$"

"a is F", where F is a predicate meaning "is identical to nothing"

The answer involves complex predicates, i.e. λ-abstraction. Intuitively, there is a distinction between "I might have failed to be human" (sentence-negation) and "I might have been not-human" (predicate-negation). Formally, where A is any formula (perhaps with free variables), the following is a *predicate*:

$\lambda x(A)$ "is such that A"

Examples:

$\lambda x({\sim}Hx)$ "is not-human"

$\lambda x(\exists y Fxy)$ "is friends-with-someone"

The following is an instance of the being constraint:

2′. $\Box(\lambda x({\sim}\exists z\, x = z)a \rightarrow \exists z\, a = z)$

But the contingentist will reject the corresponding version of line 1:

1′ $\Diamond \lambda x({\sim}\exists z\, x = z)a$

Note that accepting 1 while rejecting 1′ requires denying that $A(x)$ and $\lambda x(A(x))$ are equivalent in the sense of being interchangeable within larger subformulas.

> Contingentists are in a tricky position. If they insist that it is possible to fall under a predicate and yet be nothing, they face the charge that they are unserious about their own contingentism, because they are tacitly restricting the quantifier 'nothing' (section 4.1). If they agree that falling under a predicate entails being something, they slide into necessitism unless they distinguish not falling under a predicate from falling under a negative predicate, which is best done by means of something like the λ operator. If they introduce the λ operator, they still slide into necessitism unless they complicate its logic in awkward ways. Although none of this amounts to a refutation of contingentism, it is evidence that the view goes against the logical grain. (Williamson, 2013, p. 188)

10. Second-order logic

In first-order logic, variables can occur only in subject position:

✓ $\exists x F x$ "Something is F"

× $\exists X X a$ ["a is somehow"??]

In second-order logic, on the other hand, quantified variables can appear in predicate position too; the second sentence is grammatical.

In the language of first-order logic, you *can* say:

✓ $\exists x(Px \wedge Iax)$ "a instantiates some property"

✓ $\exists x(Sx \wedge a \in x)$ "a is a member of some set"

Each is different from saying $\exists X X a$.

There is philosophical controversy about whether second order logic is really, e.g., "set theory in sheep's clothing" as Quine said. Defenders of second-order logic have claimed that it doesn't commit you to properties or sets, and that certain paradoxes that confront first order set- and property-theory don't arise for second-order logic.

✓ $\exists x(Px \wedge {\sim} Ixx)$ "some property instantiates itself"

× $\exists X {\sim} XX$

11. Comprehension principles

Comprehension principles guarantee that a suitable array of "properties" exist. without them, second-order logic would be pointless. E.g. we couldn't apply the second-order principle of induction for arithmetic:

$$\forall X((X0 \wedge \forall n(Xn \rightarrow Xn')) \rightarrow \forall n Xn)$$

"For all properties, if 0 has that property, and if a number's successor has the property whenever that number does, then every number has the property"

In order to use the induction principle to move from "0 is an F" and "whenever n is an F, so is n's successor" to "every number is an F", we need the added premise that there is a property corresponding to the predicate F. A comprehension principle tells us this.

The usual comprehension principle in nonmodal second-order logic is this:

Comprehension $\exists X \forall x(Xx \leftrightarrow A)$

For any chosen formula A, it tells us that there is a property had by all and only things that satisfy the formula. *Note:*

- This is just the monadic case.
- This is a just schema; you get instances by replacing A with formulas.
- The formulas A can have free variables (but can't have X free).
- Instances are allowed to additionally add a "prefix" of as many universal quantifiers as you want

Some instances:

$\exists X \forall x(Xx \leftrightarrow Fx)$	"There is a property that is had by all and only the fish"
$\exists X \forall x(Xx \leftrightarrow \exists y Rxy)$	"There is a property that is had by all and only the things that respect something"
$\forall y \exists X \forall x(Xx \leftrightarrow Rxy)$	"For any y, there is a property that is had by all and only the things that respect y"

12. Williamson's argument

In second-order *modal* logic, Comprehension must be "modalized". Williamson's preferred modalization is this:

Modal comprehension $\exists X \Box \forall x(Xx \leftrightarrow A)$

This says that if you choose any formula, there is guaranteed to be a property that *necessarily* is had by all and only things that satisfy the formula.

(Aside: Modal comprehension would have false instances under Boolos's plural reading of $\exists X$:

$\exists X \Box \forall x(Xx \leftrightarrow A)$ "Some things are such that, necessarily, something is one of them iff it is a boy"

Instead, we are to understand the second-order quantifiers "intensionally"; thus "$\exists X$" is akin to "some property", not "some plurality".)

Modal comprehension implies this:

Haecceities $\forall y \Box \exists X \Box \forall x(Xx \leftrightarrow x = y)$

That is, "for every object, y, it's necessary that there exists a property that, necessarily, is instantiated by something iff that something is y"; or, more succinctly: "for every object, it's necessary that there is a haecceity of that object". Williamson then argues that contingentists ought to reject Haecceities:

> Even if I had never been, by [Haecceities] there would still have been a property tracking me (and only me). But how can it lock onto me in my absence? In those circumstances, what makes me rather than something else its target? (p. 269)

Contingentist might defend a weaker modal comprehension principle. E.g. revert to the nonmodal Comprehension, but now allow $\Box$s in the prefix.

But this is too weak; it would "[prevent] second-order logic from adequately serving the logical and mathematical purposes for which we need it" (Williamson, 2013, p. 288). E.g. it wouldn't imply:

Conjunction $\forall Y \forall Z \exists X \Box \forall x(Xx \leftrightarrow (Yx \wedge Zx))$

It would only imply the existence of "intra-world conjunctions"; it would only imply things like this:

$\Box \forall Y \forall Z \exists X \forall x(Xx \leftrightarrow (Yx \wedge Zx))$

References

Russell, Bertrand (1919). *Introduction to Mathematical Philosophy*. London: Routledge.

Williamson, Timothy (2013). *Modal Logic as Metaphysics*. Oxford: Oxford University Press.