

1. Classical physics

Dynamics Any body will accelerate in the direction of the net force on it, with a magnitude that is the ratio of the force's magnitude and the body's mass:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

Law of gravitation Each of any pair of bodies exerts a component gravitational force on the other directed toward itself, and with a magnitude that is the ratio of the product of their masses to the square of the distance between them:

$$\vec{F}_{12} = \frac{Gm_1m_2}{r^2}\hat{r}_{12}$$

The theory is deterministic (sort of) because we can calculate later positions and velocities from earlier ones using this procedure:

- Masses + positions \Rightarrow component forces (Law of Gravitation)
- Component forces \Rightarrow net forces
- Net forces + masses \Rightarrow accelerations (Dynamics)
- Accelerations + positions + velocities \Rightarrow future positions + velocities (Calculus)

2. Determinism

Laplacean Determinism A possible world w is deterministic iff for every time, t , the laws of w plus the state of w at t necessitate the state of w at any other time

What does “the state of the world at a time” mean?

- Perhaps the quantities that textbook theories invoke

- Or perhaps intrinsic (or fundamental, or intrinsic fundamental) features, whatever they in fact are

These could come apart because of, e.g., the “at-at” theory of motion, or comparativism itself.

3. Escape velocity argument

The Newtonian laws imply that a projectile will escape the gravitational attraction of a body iff its velocity exceeds its *escape velocity*, given by:

$$v_e = \sqrt{\frac{2GM}{r}}$$

(M is the mass of the body, r is its radius of the body; v_e is magnitude of the escape velocity in a direction away from the common center of mass of the body plus projectile). Note that v_e depends only on M , not on the projectile’s mass.

Now consider two possible worlds:

Earth world There exists nothing other than a planet, Earth, with a projectile whose velocity is just above Earth’s escape velocity. The projectile eventually escapes.

Pandora world Just like Earth world, except all masses are now doubled. Since the escape velocity of the planet (now called “Pandora”) is higher than Earth’s by a factor of $\sqrt{2}$, the projectile eventually returns.

Intuitive problem for determinism: the worlds have the same initial world state by comparativist’s lights, but not the same later world states.

4. At-at motion

At-at theory of motion Properties of motion are defined in terms of position over time

Easwaran’s point:

- Given the at-at theory, we must define world-states as “local” rather than intrinsic.
- But then the two worlds don’t have the same initial state, since the projectile’s acceleration affects its locations in the infinitesimal neighborhood of the original time.

5. What exactly is the argument?

To show that world w is not deterministic, we need a second world, w' , such that:

- At some time, w and w' have the same state, and at some other time, they don’t
- The laws at w are true at w'

If we’re trying to show that comparativism implies that Newtonian physics would be indeterminate, these two worlds ought to be comparativist Newtonian worlds. But Baker’s worlds are described in absolutist terms.

Since there are scale-independent differences over time between these worlds, the comparativist can certainly recognize that both Pandora’s universe and Earth’s are physically possible. But only at the expense of denying determinism. There can be no fundamental difference, for the comparativist, between the initial state of Earth’s universe and the initial state of Pandora’s. Since there is a difference between the futures of these two initial states, it must be that the same initial state can evolve into two or more distinct future states while obeying the comparativist’s laws. (Baker, 2014, p. 18)

To overcome this: First, begin with Baker’s absolutist worlds:

A-Earth: the absolutist laws of Newtonian gravitation hold; projectile escapes

A-Pandora: same laws; same initial absolute positions and velocities; absolute masses are doubled; projectile returns

Next define the *comparativist reduction* of an absolutist possible world as the corresponding comparativist’s “mosaic”. Baker’s argument then goes through, given this assumption:

Needed assumption There are two worlds, call them C-Earth and C-Pandora, whose mosaics are the comparativist reductions of A-Earth and A-Pandora, respectively, *and whose laws are the comparativist's version of the laws of Newtonian Gravitation (whatever those are)*

You can think of this as following from the assumption that *the operation of comparativist reduction transforms laws as well as mosaics.*

- Might a Humean about laws deny this, on the grounds that the C-Earth and C-Pandora worlds are too simple to allow the same laws to emerge?
- Might even a nonHumean argue that the comparativist laws are false in C-Earth and C-Pandora because their strong existence assumptions are false?

(I suspect the answer in each case is no.)

6. Representation function laws and determinism

Here's a possible way to respond to Baker. First, say that the laws concern representation functions.

One form the laws might then take would be:

For any representation functions for net force, mass, distance, and time, \vec{F} , m , d , and t , and any mapping \vec{x} from objects into \mathbb{R}^3 with corresponding distance function d , there exist real numbers k and G such that for any objects, o and o' :

$$\begin{aligned}\vec{F}(o) &= k m(o) \frac{d^2}{dt^2} \vec{x}(o) && \text{and} \\ \vec{F}(o) &= \sum_{o'} \frac{G m(o) m(o')}{d(o, o')^2} \hat{r}_{oo'} && \text{(for all objects } o')\end{aligned}$$

($\hat{r}_{oo'}$ is the unit vector pointing from o toward o' .)

This law is indeterministic, in the following sense: if you pick some representation functions to measure force, mass, distance, and time, a description of an initial state of the universe, in terms of those functions, plus the law needn't

imply the numerical description of the world at a later time, in terms of those same representation functions. (The reason is that different values of G in the absolutist's law lead to different futures, and the above law doesn't mention a specific value of G ; it only existentially quantifies it.)

But here is a different way to think about representation-function-theoretic laws. Give up on laws *simpliciter*, and instead state laws *relative to a given choice of representation functions*. So for particular representation functions for net force, mass, distance, and time, \vec{F} , m , d , and t , and any mapping \vec{x} from objects into \mathbb{R}^3 with corresponding distance function d , there will be some law, *relative to these choices*, of the form:

Representation-function Newtonian gravitation, relative to \vec{F} , m , d , and t :

For any objects, o and o' :

$$\vec{F}(o) = k m(o) \frac{d^2}{dt^2} \vec{x}(o) \quad \text{and}$$

$$\vec{F}(o) = \sum_{o'} \frac{G m(o) m(o')}{d(o, o')^2} \hat{r}_{oo'} \quad (\text{for all objects } o')$$

(Where $\hat{r}_{oo'}$ is the unit vector pointing from o toward o' .)

The following sort of determinism could then hold:

Representation-function Laplacean determinism A world is deterministic iff for any representation functions f_1, \dots at that world, a complete description of any time using $f_1 \dots$ plus the world's laws relative to those representation functions necessitates any complete description using $f_1 \dots$ of that world at any other time

What's going on is that a representation function encodes information about objects at all times. When it comes to making predictions about what a certain timeslice will lead to, a law that makes reference to such functions has more to work with than merely facts about the intrinsic features of that timeslice. It also has access, in the numerical description of that timeslice, to facts about that timeslice in relation to other timeslices.

(Baker thinks this feature of representation-theoretic laws is bad. But compare the temporal nonlocality of laws given Humeanism. No concern that we couldn't predict the future in such worlds.)

Back to the intuitive idea of what is going on. Suppose we choose representation functions for each world on which the numerical description of the initial state is in each world the same. Then the numerical constants in the corresponding law will be different. Suppose we choose representation functions so that the laws for those functions have the same constants on Pandora as on Earth. Then the functions must assign different initial values to the objects—different masses.

I think this view is quite *unappealing*, in a certain subtle and distinctive way. According to the view, the laws cannot be stated without making arbitrary choices. They're like laws that simply must be stated by reference to coordinate systems.

Baker objects to the idea that laws quantify over representation functions, by saying that it violates the idea that laws are statements about fundamental properties:

It is a familiar platitude that, while fundamentality may be a brute concept with no definition, the fundamental properties and relations “are the properties and relations that occur in the fundamental laws of physics.” (Arntzenius, 2012, p. 41) On Lewis’s popular Humean account of laws, for example, the fundamental laws are regularities in the instantiation of fundamental properties (Lewis, 1983, p. 368). And altering this feature of Lewis’s system would rob it of much of its interest. Our best theories of physics have a particular mission: to describe the universe at its most fundamental level. Insofar as they fail to do so, either through inaccuracy or by failing to describe reality in fundamental terms, we should take that as a sign that the true fundamental laws have not yet been discovered. (Baker, 2014, pp. 11–12)

This objection is not correct against the view Baker was actually considering, namely the view that the laws quantify over all representation functions. (Though the ultimate concern may be that such laws are extrinsic, in Field’s sense.)

It does at first look correct against the relativizing view, since those laws seem to be about *particular* representation functions, which are given in extension.

But really, those statements aren’t the law. What the law really is, is the *potential* to generate a relativized statement, for any choice of representation functions.

To see that there is no problem about simplicity and fundamentality, consider how things would look from a humean point of view. The humean could define the *lawhood function*—the function from choices of representation functions to laws-under-those-choices—in simple terms.

The badness of the approach is like the badness of “quotienting”, which I discuss in chapter 5 of my book. This is a kind of nomic quotienting: the world is such as to make *these* the laws under certain choices, *those* the laws under certain other choices, and so forth, but there is no further way to say what this feature of the world is.

A further objectionable feature: how to make sense of laws being counterfactually robust? A given representation function for world w has no significance for other worlds, and so the relativized law is not counterfactually robust.

7. Absolutism and Baker’s argument

It might seem obvious that Baker’s argument can’t work against absolutism, since doubling masses changes the absolutist initial condition. But it isn’t obvious that the laws will be sensitive to this change.

Mundy’s absolutism about mass The fundamental mass properties and relations consist of continuum-many determinate masses, plus two “structuring relations” over the determinate masses, \geq and $*$:

- $p \geq q$: p is at least as “large” as q
- $*(p, q, r)$: p and q “sum to” r

Given this view, are the laws representation-function-theoretic, as above with the relativizer, but with representation functions defined thus (for example):

m is a *mass function* iff for some function, M , from the determinate masses to real numbers:

- i) $M(p) \geq M(q)$ iff $p \geq q$ (for any p, q)
- ii) $M(p) + M(q) = M(r)$ iff $*(p, q, r)$ (for any p, q, r)
- iii) for any massive object, x , m assigns to x the real number assigned by M to the determinate mass property that x instantiates.

Baker's argument would then be answered, just as above. But now, facts about the future do *not* affect the world state at a time.

However, suppose the Mundyean wants intrinsic laws. Field's general approach for constructing intrinsic laws:

Step 1: convert numerical laws $f(Q_1 \dots) = g(Q'_1 \dots)$ to laws about ratios:

$$\frac{f(Q_1(x) \dots)}{f(Q_1(y) \dots)} = \frac{g(Q'_1(x) \dots)}{g(Q'_1(y) \dots)} \quad (\text{for any objects } x, y)$$

Step 2: construct intrinsic versions of the ratio laws.

For a Mundyean, the result of step 2 would concern standard sequences of properties, not concrete objects. For example, the ratio version of (the simplified version of) Newton's second law:

$$\frac{m(x)}{m(y)} = \frac{a(y)}{a(x)} \quad (\text{for any } x, y)$$

would become:

Absolutist Intrinsic-Newton For any objects x and y , with determinate acceleration and mass properties a_x, a_y, m_x, m_y , there do *not* exist a mass sequence S_1 and an acceleration sequence S_2 such that i) $m_x \in S_1$; ii) $m_x \in S_2$; iii) there are exactly as many members of S_1 that are $\leq_m m_x$ as there are members of S_2 that are $\leq_a m_y$; and iv) there are fewer members of S_2 that are $\leq_a m_x$ than there are members of S_1 that are $\leq_m m_y$; and there do *not* exist an acceleration sequence S_1 and a mass sequence S_2 such that i) $m_y \in S_1$; ii) $m_y \in S_2$; iii) there are exactly as many members of S_1 that are $\leq_a m_y$ as there are members of S_2 that are $\leq_m m_x$; and iv) there are fewer members of S_2 that are $\leq_m m_y$ than there are members of S_1 that are $\leq_a m_x$

where acceleration and force sequences are now sequences of determinate properties, not concrete objects.

Now for an argument that even given Mundyean absolutism, Field's approach to intrinsic laws leads to indeterminism. The argument is based on:

Mass doubling If a Mundyean mosaic obeys Mundyean intrinsic laws, then so does any other mosaic that differs only by all masses being doubled

This is true because doubling mass does not affect whether ratio versions of laws about mass are true. Consider Field's versions, for example:

Law of gravitation:

...at any point the Laplacean of the gravitational potential is proportional to the mass-density at that point... (Field, 1980, p. 78)

Dynamics:

the acceleration of a point-particle subject only to gravitational forces is at each point on the particle's trajectory equal to the gradient of the gravitational potential at that point. The invariant content of this law is exhausted by the claim that the gradient is proportional to the acceleration... (Field, 1980, p. 81)

The argument

1. Begin with Baker's flat-footed absolutist worlds:

A-Earth: the absolutist laws of Newtonian gravitation hold; projectile escapes

A-Pandora: same laws; same initial absolute positions and velocities; all absolute masses are doubled; projectile returns

2. Construct "Mundyean versions" of these worlds:

M-Earth: Mundyean version of A-Earth; projectile escapes. (Intrinsic Mundyean laws hold.)

M-Pandora: Mundyean version of A-Pandora: all masses are doubled; projectile returns. (Intrinsic Mundyean laws hold.)

3. Apply the principle of Mass doubling to M-Earth, to yield:

Doubled M-Earth: Like M-Earth, but with all masses doubled. Projectile escapes. (Intrinsic Mundyean laws hold.)

Given M-Pandora, determinism fails at Doubled M-Earth.

References

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