QUANTITY

Ted Sider
Structuralism seminar

1. The problem of quantity

What metaphysics of mass and other quantitative properties accounts for the use of numbers in science to measure those properties?

Flat-footed metaphysics of quantity Quantitative scientific theories are about relations between concrete objects and numbers, such as the mass-in-kilograms relation, the mass-in-grams relation, and so forth.

Perhaps fine under modal+ontological tools. Could supplement with:

> Necessarily, for any object x and real number y, mass-in-kilograms(x, y) iff mass-in-grams($x, 1000y$)
>
> Necessarily, for any objects x, x' and real numbers y, y', if mass-in-kilograms(x, y) and mass-in-kilograms(x', y'), then: $y > y'$ iff x is more massive than y

But given the tool of concept-fundamentality, we must ask: *which* of these mass relations are fundamental?

- Arbitrary to pick just one (privileges a unit)
- Redundant to include all.

2. Simple absolutism

Simple Absolutism the "determinate masses" are the only fundamental mass properties or relations

No privileged units; but no account of numerical measurement.

3. Comparativism

Comparativism (about mass) The relations $\succeq$ and C are the only fundamental mass properties or relations

- $x \succeq y$: x is at least as massive as y
- $Cxyz$: x and y's combined masses equal z's

(The relations are not defined in terms of underlying numerical scales.)

4. Measurement theory

- A mathematical investigation of relationships between numerical and nonnumerical representations of quantities. Proves theorems like these:

Representation theorem There exists at least one mass function

Uniqueness theorem Any two mass functions, g and h, are scalar multiples—i.e., for some positive real number a, $g(x) = a \cdot h(x)$ for all individuals x

where a *mass function* is a function f from individuals to real numbers such that i) $f(x) \geq f(y)$ iff $x \succeq y$ and ii) $f(x) + f(y) = f(z)$ iff $Cxyz$

4.1 Coding up nonnumerical facts

These theorems show how numerical and nonnumerical facts about quantities are "correlated". For example, suppose y is twice as massive as x. Nonnumerical fact:

$$Cxxy \tag{T}$$

This is correlated with the numerical fact $k(y) = 2 \cdot k(x)$, since that implies:

$$k(x) + k(x) = k(y) \tag{$\mathrm{T}_{\mathbb{R}}$}$$

which implies (T) if k is a mass function.

This argument is not tied to the particular mass function k we choose. For any other mass function f, given the uniqueness theorem, for some real number a, $k(x) = a \cdot f(x)$ and $k(y) = a \cdot f(y)$. So ($\mathrm{T}_{\mathbb{R}}$) implies

$$a \cdot f(x) + a \cdot f(x) = a \cdot f(y)$$

and so

$$f(x) + f(x) = f(y)$$

which implies $Cxxy$ as before.

In general, any two mass functions g and h agree on the ratios of masses:

$$\frac{g(x)}{g(y)} = \frac{a \cdot h(x)}{a \cdot h(y)} = \frac{h(x)}{h(y)}$$

So numerical features determined by ratios don't turn on the chosen mass function. Other numerical features do, e.g., *being assigned an even integer*. These features are "artifacts" of particular mass functions, and don't code up nonnumerical facts.

4.2 Assumptions needed for representation and uniqueness theorems

Assumptions are needed to prove representation and uniqueness theorems. Some typical assumptions (where "$x \succ y$" means that x is *more* massive than y, i.e., $x \succeq y$ but $y \not\succeq x$):

Transitvity of $\succeq$ If $x \succeq y$ and $y \succeq z$ then $x \succeq z$

Monotonicity If $x \succeq y$ and $C(x, z, x')$ and $C(y, z, y')$, then $x' \succeq y'$

Positivity if $C(x, y, z)$ then $z \succ x$

Existence of sums For any x and y there is some z such that $Cxyz$

Density If $x \succ y$ then for some z, $x \succ z \succ y$

5. Comparativism as structuralism

6. Laws and fundamentality

The problem for the simple absolutist may now be sharpened: it cannot recognize simple laws. It can only recognize:

> "if a particle has exactly *this* mass and experiences exactly *that* force, then it will undergo exactly *such-and-such* acceleration; but if it has exactly this other mass and experiences exactly that other force, then it will undergo exactly thus-and-so acceleration; and if…".

7. Comparativism and existence assumptions

Comparativists normally make strong "existence assumptions", e.g. Existence of sums and Density. What exactly goes wrong if they're false?

- Representation theorem still holds.
- Uniqueness theorem false, but that might be harmless.

But what kinds of laws will be possible?

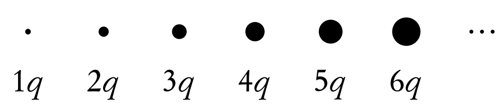
8. Existence assumptions and intrinsic laws

One sort of law would be laws that quantify over representation functions.

Representation-function-Newton There exist simple representation functions for force, mass, and acceleration; and for any simple representation functions for those quantities, f, m, and a, there exists some real number k such that for any object x, $f(x) = km(x)a(x)$

where "simple" representation functions are those that satisfy simple numerical laws. But could there be "intrinsic" laws in Field's (1980) sense? Field's approach to comparativist laws uses "standard sequences".

S is a Q *sequence* $=_{\text{df}}$ S is a set containing some member, s_1, such that for every $x \in S$, there is some $y \in S$ such that: a) $C_Q s_1 x y$, and b) for any $z \in S$, if $y \succeq_Q z \succeq_Q x$ then $z = x$ or $z = y$



A Q sequence is a "grid" one can lay down on all objects; objects can be measured as integer multiples of the grid's "unit"—the Q value of its first member—with accuracy that increases as the grid's resolution increases. Field's method for constructing intrinsic laws uses quantification over grids with arbitrarily high resolution. If the existence assumptions fail, the grids won't exist, and Field's proposed intrinsic laws will either be false or else vacuously true and hence too inferentially weak to be laws.

For example, pretend that acceleration is a primitive scalar quantity taking only positive values (so that it's a ratio scale), and that all objects undergo exactly the same net force, so that $F = ma$ says

$$m(x)a(x) = m(y)a(y) \qquad \text{(for any } x, y\text{)}$$

The Fieldian intrinsic correlate is then:

Intrinsic Law For any objects x and y: there do *not* exist a mass sequence S_1 and an acceleration sequence S_2 such that i) $x \in S_1$; ii) $x \in S_2$; iii) there are exactly as many members of S_1 that are $\preceq_m x$ as there are members of S_2 that are $\preceq_a y$; and iv) there are fewer members of S_2 that are $\preceq_a x$ than there are members of S_1 that are $\preceq_m y$; and there do *not* exist an acceleration sequence S_1 and a mass sequence S_2 such that i) $y \in S_1$; ii) $y \in S_2$; iii) there are exactly as many members of S_1 that are $\preceq_a y$ as there are members of S_2 that are $\preceq_m$ than x; and iv) there are fewer members of S_2 that are $\preceq_m y$ than there are members of S_1 that are $\preceq_a x$

This can be true simply because of the nonexistence of appropriate mass or acceleration sequences. In such circumstances, "Intrinsic Law" won't be a *law* because it is too inferentially weak. It doesn't have the consequences about C_m, C_a, $\succeq_m$, and $\succeq_a$ that ∃-Newton has, for example.

(For example, if $C_m xxy$, then y's mass is twice that of x for any mass measurement function, and so ∃-Newton implies that y's acceleration is half x's, and so $C_a yyx$. But Intrinsic Law doesn't have this consequence in the absence of the existence assumptions, since there are models in which $C_m xxy$ holds and $C_a yyx$ doesn't, and yet Intrinsic Law holds simply because of the absence of mass and acceleration sequences.)

9. Intrinsic laws and Mundy

Mundy's (1989) primitive multigrade predicate:

$a_1, \ldots, a_n \succeq b_1, \ldots, b_m$ (for any finite numbers of arguments n, m)

"$a_1, \ldots, a_n$ together have a sum total of mass that is at least as great as the sum total of mass possessed by $b_1, \ldots, b_m$"

Standard ways of saying that $\frac{m(x)}{m(y)} \geq \frac{n}{m}$ rely on existence assumptions, e.g.:

> There exist $x_1, x_2, \ldots x_{n-1}$ and $y_1, y_2, \ldots, y_{m-1}$ such that i) $Cx_1x_1x_2$, $Cx_1x_2x_3$, …, $Cx_1x_{n-1}x$; ii) $Cy_1y_1y_2$, $Cy_1y_2y_3$, …, $Cy_1y_{m-1}y$; and iii) $x_1 \succeq y_1$
>
> $$\underbrace{\bullet \quad \bullet \quad \ldots \quad \bullet \quad \bullet \quad x}_{n \text{ evenly spaced things}} \qquad \underbrace{\bullet \quad \bullet \quad \ldots \quad \bullet \quad \bullet \quad y}_{m \text{ evenly spaced things}}$$

Mundy's does not:

$$\underbrace{x, \ldots, x}_{m \text{ occurrences}} \succeq \underbrace{y, \ldots, y}_{n \text{ occurrences}}$$

Mundy is able to prove representation and uniqueness theorems without existence assumptions, but not formulate intrinsic laws. His correlate of "$\frac{m(x)}{m(y)} \geq \frac{n}{m}$" is for *fixed* n and m; but for intrinsic laws about ratios of real-valued quantities we need n and m to be variable, as in "for any integers n and m, $\frac{m(x)}{m(y)} \geq \frac{n}{m}$ iff $\frac{m(u)}{m(v)} \geq \frac{n}{m}$".

10. Intrinsicality of laws

Why think that laws should be intrinsic? (Even extrinsic laws could still be simple and strong constraints on the fundamental concepts.)

Field's complaints about extrinsic laws:

"Causally irrelevant" entities

> If, as at first blush appears to be the case, we need to invoke some real numbers like 6.67×10^{-11} (the gravitational constant in $m^3/kg^{-1}/s^{-2}$) in our explanation of why the moon follows the path that it does, it isn't because we think that that real number plays a role as a cause of the moon's moving that way. (Field, 1980, p. 43)

But fundamental extrinsic laws wouldn't imply that we can see or touch real numbers, that purely numeric facts about real numbers (such as that $3 = 2 + 1$) cause or are caused by physical facts such as that I am sitting, am in a certain location, am more massive than a mouse, etc.

"Extraneous" entities

- Representation-function-Newton quantifies over mathematical entities, which aren't, intuitively, part of the proper subject matter of the laws of motion.
- But aren't Field's standard sequences also "extraneous"? (Melia, 1998, section 2)
- Further (related) concern: nonlocality

"Arbitrariness"

> …one of the things that gives plausibility to the idea that extrinsic explanations are unsatisfactory if taken as *ultimate* explanation is that the functions invoked in many extrinsic explanations are so arbitrary" (Field, 1980, p. 45)

- But extrinsic laws like Representation-function-Newton don't make arbitrary choices (it quantifies over representation functions)
- Nor would this variant on the flat-footed view: the fundamental mass-concept is a relation between a pair of objects and a number representing the ratio between their masses.

My overall concern: what's worrisome about extrinsic laws is some combination of arbitrariness and artificiality; but is there any way to avoid this? The intuitive idea of the concept-fundamentalist's epistemology is that the laws ought to look attractive as laws when they're viewed as they fundamentally are—when they're formulated in a completely fundamental language. The concern is that they just won't look good when viewed in that way.

Mixed absolutism (Mundy, 1987) the determinate masses, plus two higher-order "structuring relations" over the determinate masses, $\geqslant$ and $*$, are the only fundamental mass properties or relations

where the structuring relations—"second-order" counterparts of $\succeq$ and C—may be glossed as follows:

- $p \geqslant q$: p is at least as "large" as q
- $*(p,q,r)$: p and q "sum to" r
- (Arntzenius and Dorr (2011) defend a related view, but with points of substantival "quality-spaces" replacing properties).
- Even here there is no escape from arbitrariness/artificiality/extraneous entities.
- Though at least we have (spatiotemporal) locality.

11. Fundamentality versus ground

We have framed the issue in terms of fundamentality, which made the search for intrinsic laws central. Shamik Dasgupta (2013, 2014) frames the issue in terms of ground, and his defense of comparativism focuses on entirely different issues. A further illustration of the importance of the question of tools.

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