From Newton to Minkowski

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Philosophy of Time

1. Newtonian space + time

In order for the spatiotemporal notions in his laws, e.g.,

Law I Every body perseveres in its state either of rest or of uniform motion in a straight line, except insofar as it is compelled to change its state by impressed forces

Law II The change in motion is proportional to the impressed motive force and is made along the straight line on which the force is impressed

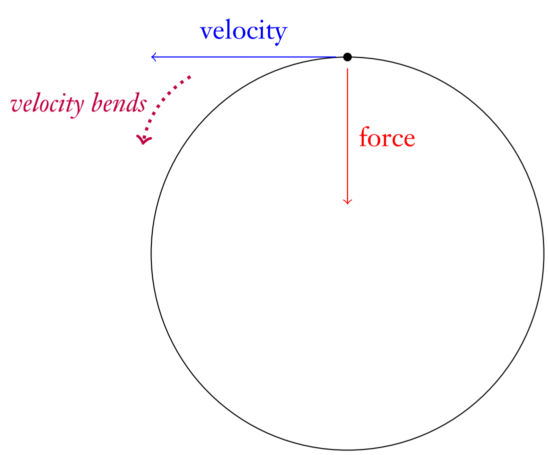
to be well-defined, Newton adopts this conception of spatiotemporal structure:

Newtonian space + time:

1. There are moments of time, and enduring points of space
2. Time is one-dimensional and has a metric
3. Space is three-dimensional, and has a (Euclidean) metric
4. Material bodies are located at points of space at times

The bottom line is that for Newton, motion is change in *absolute position*.

Change in "motion", or velocity, includes change in direction as well as speed, as is illustrated by a ball whirled in a circle on a string:



2. The bucket argument

Newton argued for his view by considering the meniscus on the surface of water in a spinning bucket. (Or the elongation of a spring connecting two rotating globes.) There is no relevant *relative* motion, so the motion must be with respect to absolute space.

This was a problem for Leibniz, who rejected substantivalism about space, and hence thought that all motion is relative to other bodies.

3. The undetectability argument

The bad news for Newton (according to some) is that his absolute velocities (i.e., velocity with respect to absolute space) are undetectable.

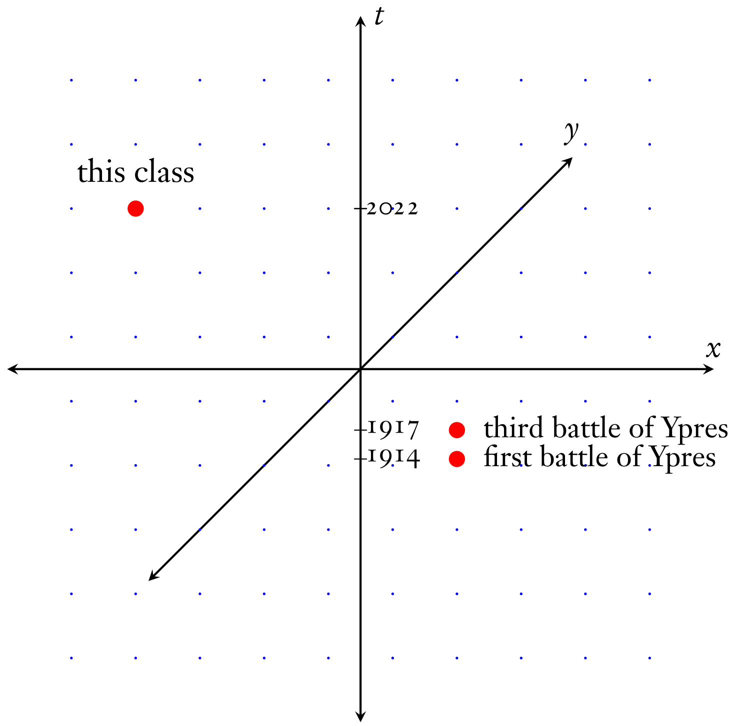
We can't *directly* observe absolute velocity. All we directly observe is *relative* velocity, like how fast my car is moving relative to the surface of the earth. And we can't have indirect evidence for what something's absolute velocity is, since Newton's laws imply that any physical system will behave the same in observable ways if given a "velocity boost".

But since we *need* absolute *acceleration* (because of the bucket argument), aren't we *stuck* with absolute velocities? Surprisingly, as we now know, no.

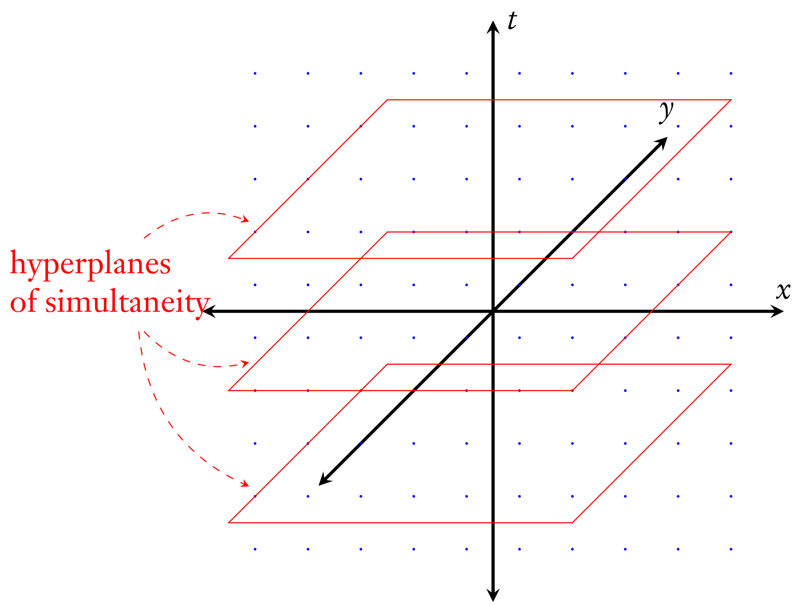
4. Spacetime

Step one: replace space+time with *spacetime*. Points of spacetime are "place-times"; locations of possible instantaneous events.

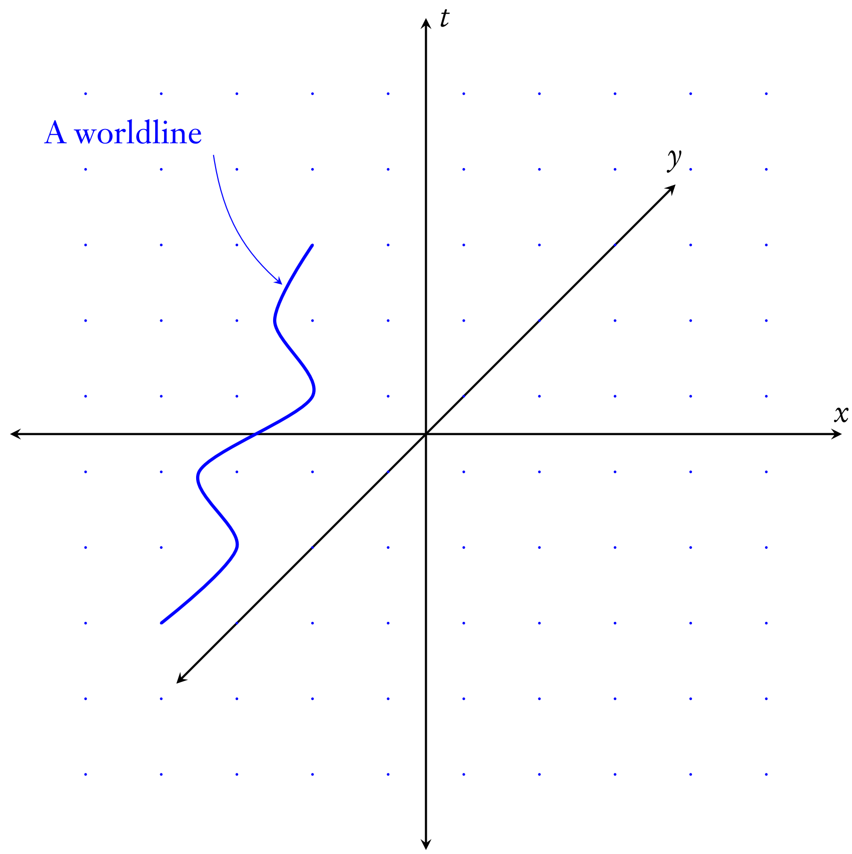
Like space, spacetime has structure. E.g., maybe it's four-dimensional, one of the dimensions being time, along which temporal distances are well-defined:



A maximal collection of simultaneous points is a “hyperplane of simultaneity”:



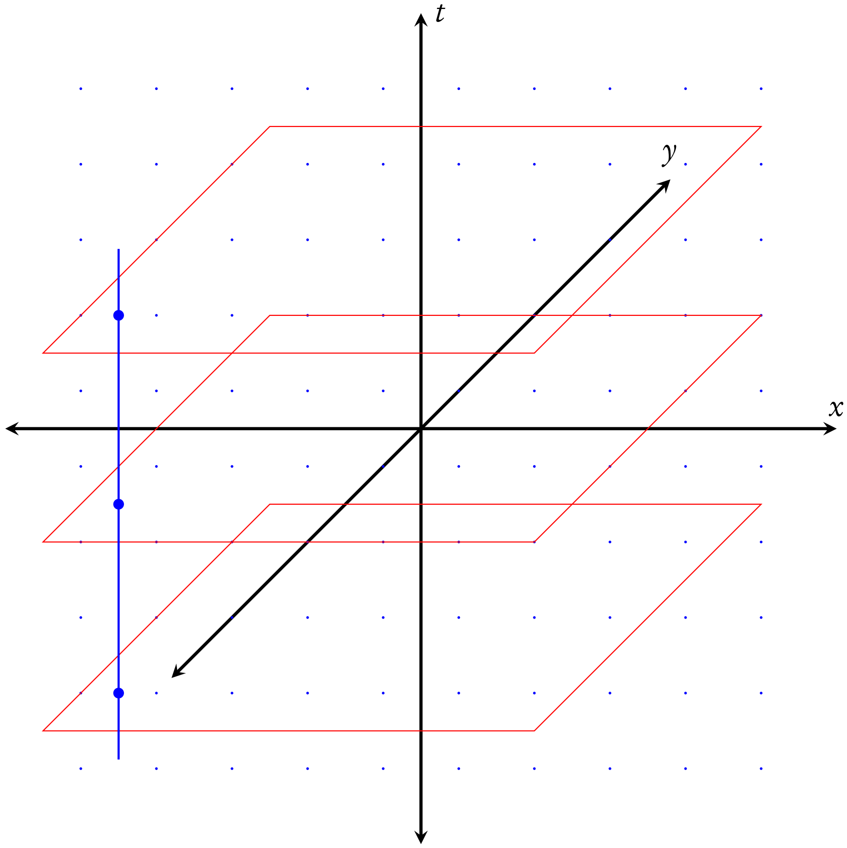
In spacetime, bodies occupy points simpliciter (rather than relative to times). The set of points an object occupies is its “worldline”:



5. Newtonian spacetime

1. There are points of spacetime
2. There is a temporal distance metric defined over all points
3. Within any hyperplane, there is a Euclidean distance metric
4. There is a same-place-as relation, holding connecting each point to exactly one point in each hyperplane
5. Material bodies are located at points in spacetime

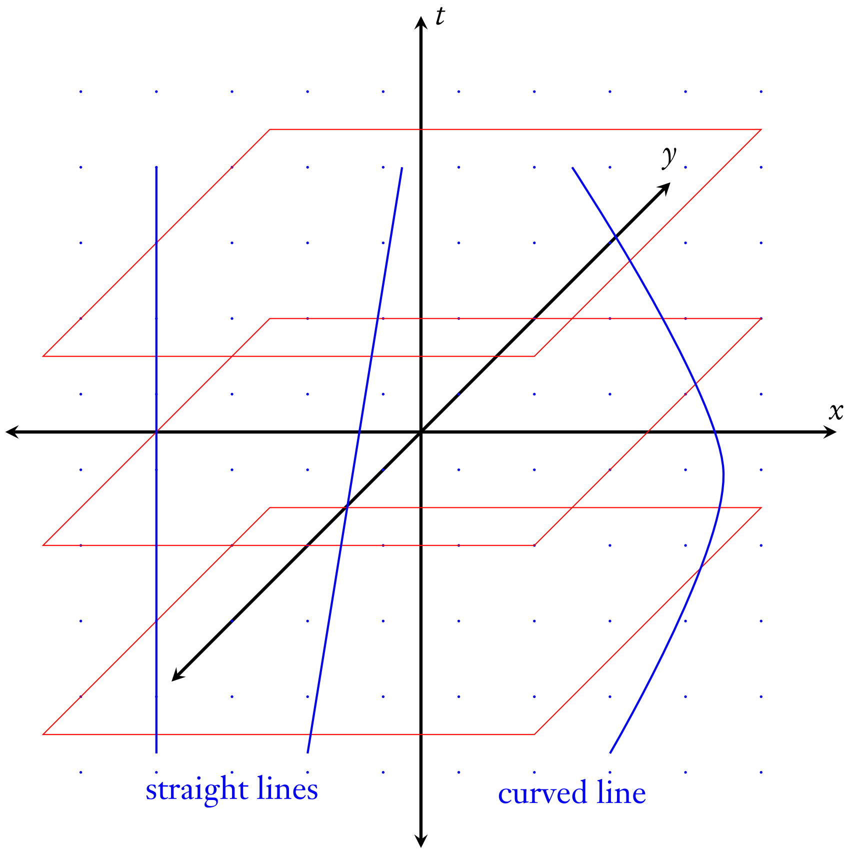
A worldline in which all the points are at-the-same-place-as one another:



6. Galilean spacetime

Begin with Newtonian spacetime, but then:

1. Delete the same-place-as relation
2. Add an affine structure for the whole spacetime (straight lines intersecting multiple hyperplanes)



We can rewrite the laws of motion so that they make sense in this spacetime:

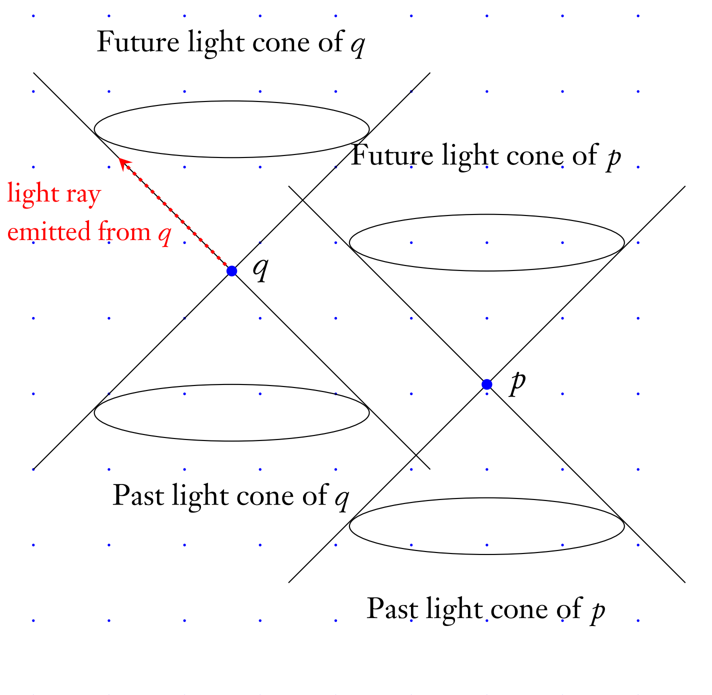
Law I (revised) Every body follows a straight line through spacetime, except insofar as it is acted on by impressed forces

Law II (revised) The curvature in a worldline is proportional to the impressed force and is in direction in which the force is impressed ($F = m\mathbf{Bend}$)

Now only *relative* velocities make sense, not absolute velocities. But absolute acceleration does make sense: it is the degree to which a worldline is curved.

7. Minkowskian spacetime

1. Same topological and affine structure as Galilean spacetime
2. From each point, there is a distinguished pair of classes of points: the past and future "light cones" of that point

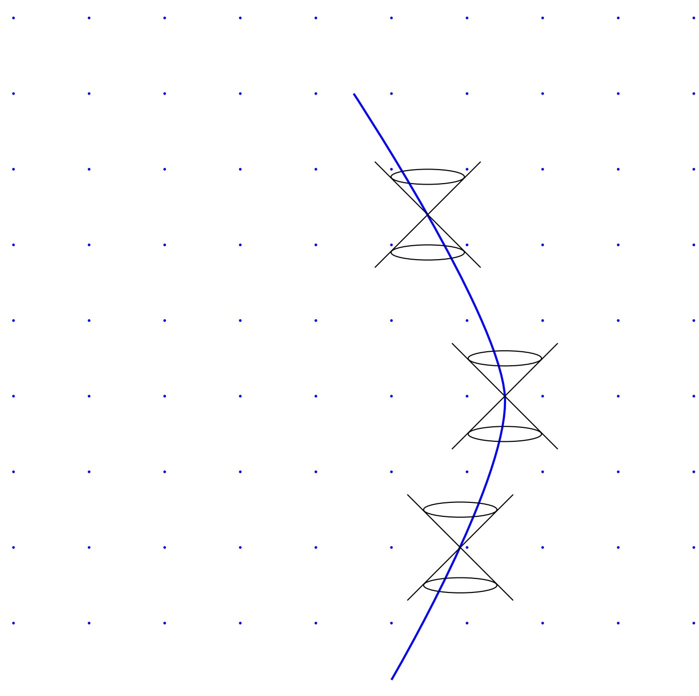


Law of Light The trajectory of a light ray emitted from an event (in a vacuum) is a straight line on the future light-cone of that event.

> If a flashbulb goes off at [a point] (in a vacuum) the light, spreading out in all directions, will occupy the surface of its entire future light-cone. Intuitively, in three-dimensional space, a flashbulb produces an expanding sphere of light. With one spatial dimension suppressed, this becomes an expanding circle of light, which appears as a cone in our diagram. (Maudlin, p. 73)

Limiting Role of the Light-cone The trajectory of any physical entity that goes through an event never goes outside the light-cone of that event.

Relativistic Law of Inertia The trajectory of any physical entity subject to no external influences is a straight line in Minkowski space-time.



Minkowski spacetime continued:

3. No well-defined notion of simultaneity
4. Metric: the relativistic "interval"

Reference frame A line that stays within light cones

Frame-relative simultaneity Suppose you are moving in frame F, you bounce a light ray off a mirror at e', and you detect its return. The event e at the midpoint (as measured by the interval), along F, between the emission of the lightray and its return, is simultaneous with e relative to F.