Diamonds are Forever

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Philosophy of Time

1. Modal tense logic

The language of modal tense logic includes both modal and tense operators. How do they interact?

At the very least, logical truths (including modal- and tense-logical truths) hold permanently and necessarily. Thus, e.g.:

$$\Box(p \to \mathsf{S}p)$$

$$\mathsf{A}(p \to \Diamond\Box p)$$

But that leaves a lot open, e.g.:

Perpetuity Necessary truths are permanent

$$\forall p(\Box p \to \mathsf{A}p)$$

The interesting question is whether Perpetuity is correct assuming:

Propositional temporalism Some propositions are only temporarily true

$$\exists p(p \wedge \sim\mathsf{A}p)$$

Dorr and Goodman (2020) say that it is.

2. An intuitive, semantic account of the issue

Kripke semantics for modal logic

A set of worlds

Sentences have truth values relative to worlds

$\Box\phi$ is true at a world iff ϕ is true at every world
$\Diamond\phi$ is true at a world iff ϕ is true at some world

(Modal operators shift the world parameter)

Kripke semantics for tense logic

A set of times

Sentences have truth values relative to times

$\mathsf{A}\phi$ is true at a time iff ϕ is true at every time
$\mathsf{S}\phi$ is true at a time iff ϕ is true at every time

(Tense operators shift the time parameter)

Kripke semantics for modal tense logic (partial)

A set of worlds

A set of times

Sentences have truth values relative to world-time pairs

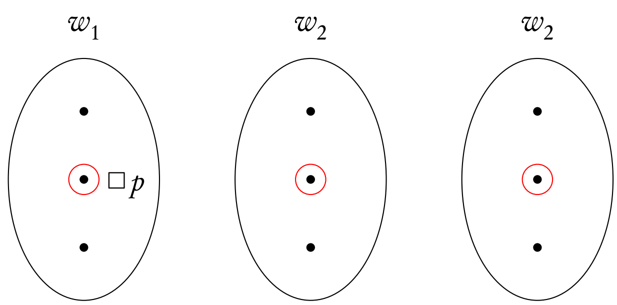
$\mathsf{A}\phi$ is true at $\langle w, t\rangle$ iff for every t', ϕ is true at $\langle w, t'\rangle$
$\mathsf{S}\phi$ is true at $\langle w, t\rangle$ iff for some t', ϕ is true at $\langle w, t'\rangle$

(Tense operators shift just the time parameter)

What about modal operators? Two main views:

Fine (1977)/Kaplan (1979) truth conditions

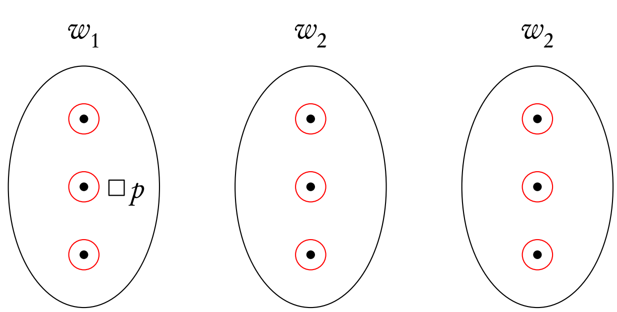
$\Box\phi$ is true at $\langle w, t\rangle$ iff for every w', ϕ is true at $\langle w', t\rangle$
$\Diamond\phi$ is true at $\langle w, t\rangle$ iff for some w', ϕ is true at $\langle w', t\rangle$
(Modal operators shift just the world parameter)



p must be true at all the circled points

Montague (1973) truth conditions

$\Box\phi$ is true at $\langle w, t\rangle$ iff for every w' and every t', ϕ is true at $\langle w', t'\rangle$
$\Diamond\phi$ is true at $\langle w, t\rangle$ iff for some w' and some t', ϕ is true at $\langle w', t'\rangle$
(Modal operators shift both parameters)



p must be true at all the circled points

3. The underlying issue: Symmetry

What claim in the language of modal tense logic (as opposed to a claim about Kripke semantics) is central for opponents of Perpetuity?

Symmetry Every falsehood necessitates something that is never true when it is

$$\forall p\Big(\sim p \to \exists q\big(\Box(p \to q) \wedge \mathsf{A}(p \to \sim q)\big)\Big)$$

Equivalent claim:

Supervenience Every truth is necessitated by a permanent truth

$$\forall p\Big(p \to \exists q\big(\Box(q \to p) \wedge \mathsf{A}q\big)\Big)$$

$$h \text{ is a } \textit{history} =_{\text{df}} \mathsf{A}h \wedge \forall q\big(\mathsf{A}q \to \mathsf{A}\Box(h \to q)\big)$$

$$s \text{ is a } \textit{snapshot} =_{\text{df}} s \wedge \forall p\big(p \to \mathsf{A}\Box(s \to p)\big)$$

A history might take this form:

$$h = \mathsf{S}(\ldots \mathsf{P}_2 s_{-2} \wedge \mathsf{P}_1 s_{-1} \wedge s_0 \wedge \mathsf{F}_1 s_1 \wedge \mathsf{F}_2 s_2 \wedge \ldots)$$

where the s_is are "sometimes-snapshots".

For each possible history, h, there is exactly one of its possible snapshots, s, such that $\Box(h \to s)$. Form a conjunction of conditionals from possible histories to their entailed snapshots:

$$(h \to s) \wedge (h' \to s') \wedge \ldots$$

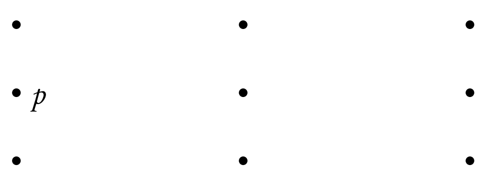
This is the analog, if we aren't reifying times, of "it is time t" (since $s, s', \ldots$ describe what is going on at the present moment, in all possible histories). It is necessarily true.

> But the falsity of NOW also causes trouble for those who accept *Symmetry* on some other grounds, and thus reject *Perpetuity*. Consider the question: which possibly sometimes-true propositions are possibly true? According to proponents of *Perpetuity*: all of them. According to proponents of NOW: those that are possibly true *now*. But what about people who reject both *Perpetuity* and NOW? They must think that some but not

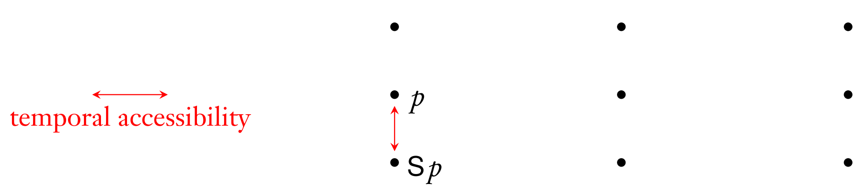
> all propositions that could have been sometimes true but could not have been true now are possibly true. If they accept (the necessary eternal truth of) *Symmetry*, they must also think that, for each possible world-history, there is a unique time in that history which could have been present had that history obtained. Given the falsity of NOW, this function from possible world-histories to members of their respective time-series cannot be the constant function that maps every history to the present time. But it is hard to see how this function could then fail to draw arbitrary distinctions of a sort that ought to disqualify it from marking the boundaries of metaphysical possibility. (p. 655)

Dorr and Goodman's method for isolating Symmetry: develop a general conception of models, of which Fine/Kaplan and Montague models are special cases. Find an abstract characterization of Fine/Kaplan models. (Models in which each point is "unaccompanied" and "square-completing".) Find sentences corresponding to this characterization. One of those sentences is Symmetry.

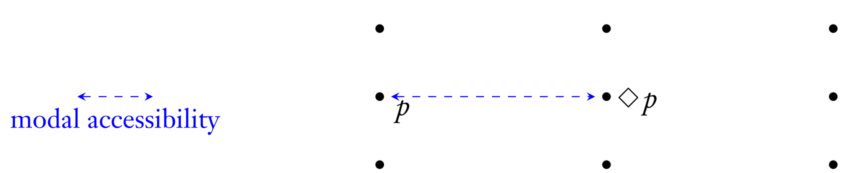
"Points" in these models, at which we evaluate formulas, are structureless:



A tensed statement makes demands on *temporally accessible* points:

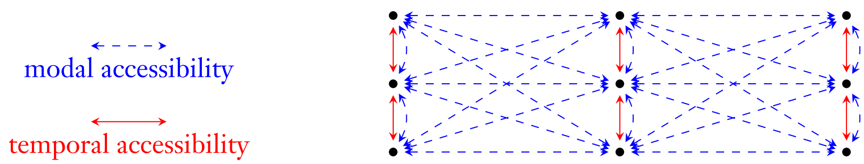


A modal statement makes demands on modally accessible points:



Picture of modal and temporal accessibility for the Montague approach:

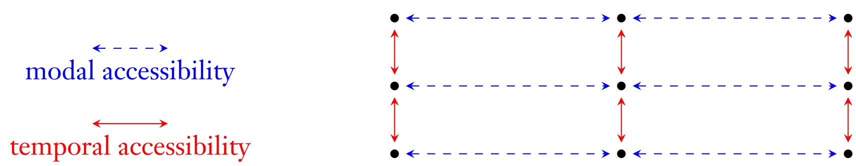
Montague



Time-modality asymmetry: temporal equivalence classes are proper subsets of modal equivalence classes.

Picture of temporal accessibility for Fine/Kaplan:

Fine/Kaplan



Symmetry restored: *modal and temporal equivalence classes intersect only in a single point*. This is the condition to which Symmetry corresponds.

Now set aside semantics. Symmetry plus Perpetuity implies Propositional Eternalism:

1.	$\sim p$	(choose p to be any false proposition)
2.	$\exists q\big(\Box(p \to q) \wedge \mathsf{A}(p \to \sim q)\big)$	1, Symmetry
3.	$\forall q\big(\Box(p \to q) \to \mathsf{A}(p \to q)\big)$	Perpetuity
4.	$\mathsf{A}{\sim}p$	

4. Argument against Perpetuity: NOW

Dorr and Goodman show that the following principle implies Symmetry:

NOW Every proposition is, necessarily, true just in case it is now true

$$\forall p \Box (p \leftrightarrow \mathsf{N} p)$$

"N" is a sentence operator meaning "it is now the case that".

Argument for NOW:

1. $\forall p \Box \mathsf{N}(p \leftrightarrow \mathsf{N} p)$ (premise)
2. So, $\forall p \mathsf{N} \Box (p \leftrightarrow \mathsf{N} p)$ ("$\Box\mathsf{N} \Rightarrow \mathsf{N}\Box$")
3. So, $\forall p \Box (p \leftrightarrow \mathsf{N} p)$ ($Nq \Rightarrow q$)

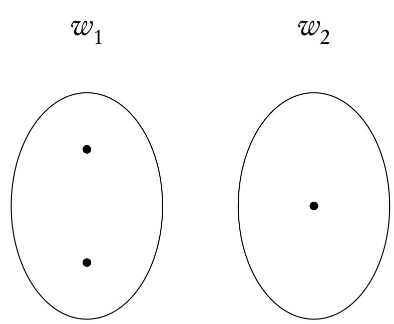
Dorr and Goodman reject the move from 1 to 2. From their point of view:

> "Could it have now been the case that: dinosaurs roam without roaming now? No. Is it now the case that dinosaurs could have roamed without roaming now? Yes!"

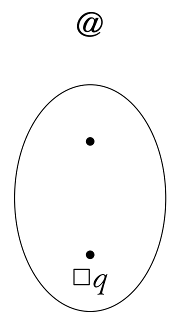
They say that our intuitions about the inference are unreliable since, given Perpetuity, 'now, Necessarily' is weird to utter if 'necessarily' means metaphysical necessity (since the 'now' would be redundant).

5. Argument for Perpetuity: contingent cardinality of the time series

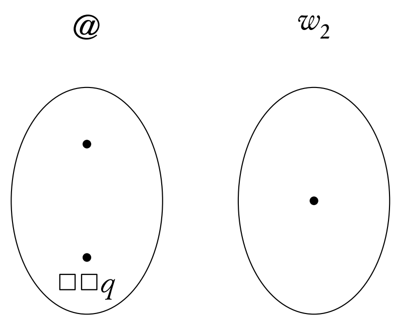
Suppose there could have been only one time:



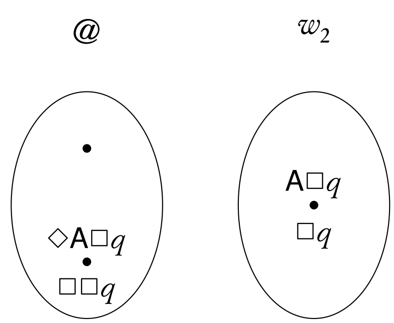
Which time in w_1 is the same as the one time in w_2?



1. $\Box q$ (suppose)
2. $\Box\Box q$ (1, by the "4" principle of modal logic)
3. $\Diamond\forall p(p \to \mathsf{A}p)$ "there could have been just one moment"



4. $\Diamond\mathsf{A}\Box q$ (2,3, by "K" modal reasoning)



5. $\mathsf{A}\Diamond\Box q$ (4, by the "Church-Rosser" principle)
6. $\mathsf{A}q$ (5, by the "B" principle of modal logic)

Church-Rosser

Whatever could always be true, always could be true

$\forall p(\Diamond \mathsf{A} p \to \mathsf{A} \Diamond p)$

Argument:

i) Any proposition of the form $\mathsf{A}p$ is eternal

ii) If a proposition p is eternal, so is $\Diamond p$

iii) Therefore, $\Diamond \mathsf{A} p$ is eternal

iv) Therefore, $\Diamond \mathsf{A} p$ can't change its truth value; if $\Diamond \mathsf{A} p$ then $\mathsf{A} \Diamond \mathsf{A} p$

v) And so, by the "T" principle of modal logic, if $\Diamond \mathsf{A} p$ then $\mathsf{A} \Diamond p$

Idea behind ii): possibility and necessity aren't "sources of temporariness".

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