Spatial structure

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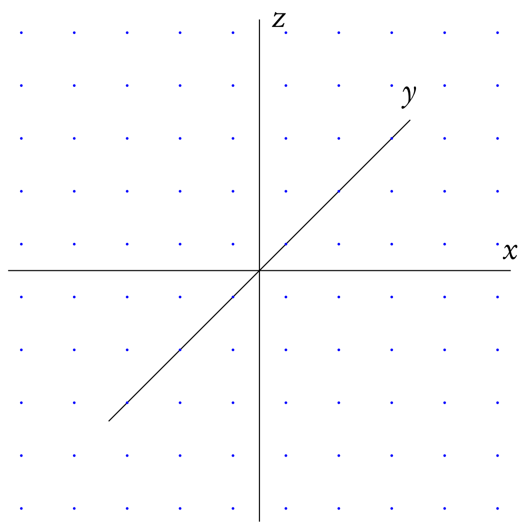
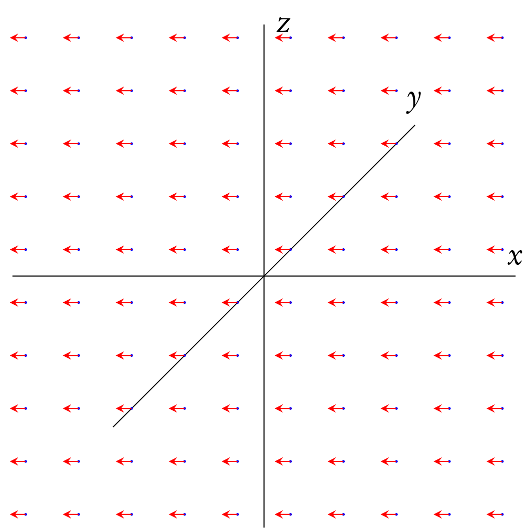
Philosophy of Time

1. The idea of spatial structure

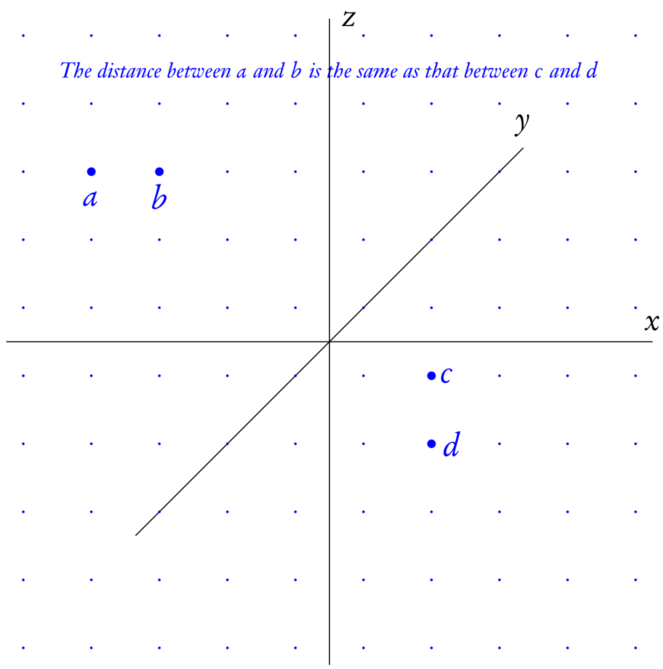
Left and *right* are "relative" to one's orientation. They aren't "built into" space itself; they aren't "intrinsic" or "absolute". Absolute left and right aren't "well-defined". Space's intrinsic structure looks like this:



not like this:



What *is* built in? Distance, for one (presumably):



The question of the structure of space is in part the question of what spatial features are "intrinsic" ("absolute", etc.)

(Also it is about what laws intrinsic features obey. E.g., are distances Euclidean?)

2. Laws and spatial structure

Common assumption: laws of nature can only make reference to intrinsic spatial structure. E.g., there couldn't be a law saying:

> "all negatively charged particles travel to the left"

(Unless there really was a distinguished direction *left*!)

Aristotle thought there was a law saying that "Earth tends to move down, and fire tends to move up". Accordingly, space according to him had distinguished directions of up and down.

Newton, also, accepted the spatial structure needed for his laws to make sense.

Law I Every body perseveres in its state either of rest or of uniform motion in a straight line, except insofar as it is compelled to change its state by impressed forces.

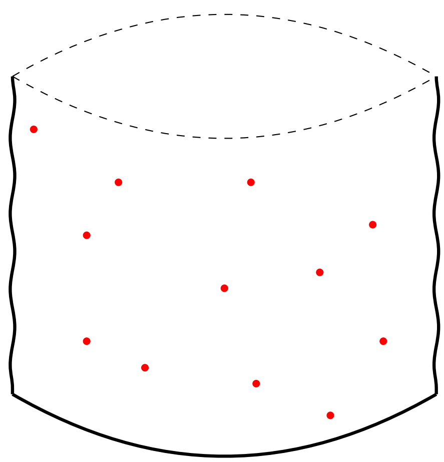
Makes reference to: *being at rest*, and *moving in a straight line*, each of which is well-defined in Newton's space and time.

3. Kinds of spatial structure

Varieties of spatial structure come in "levels".

3.1 Set-theoretic structure

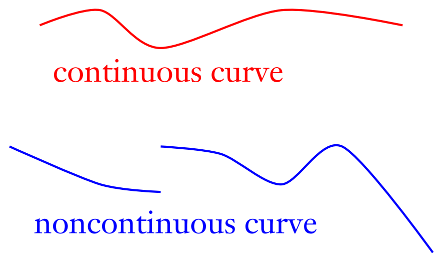
In a "bag of points", all that is significant is the *number* of points:



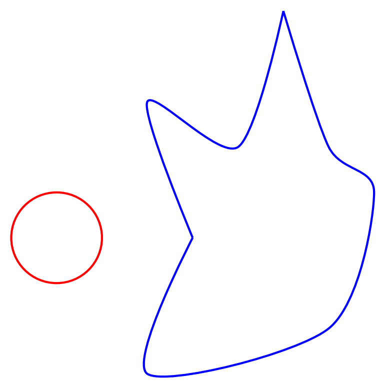
This isn't really a "space" since it has no spatial structure; it's a "mere set".

3.2 Topological structure

Involves notions like *continuity*:



Not size or shape or straightness. These figures are topologically the same:



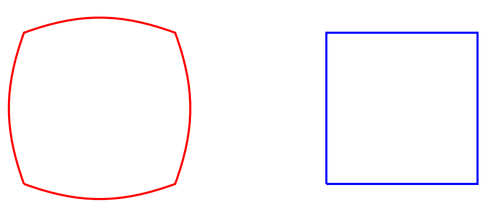
3.3 Differentiable structure

Involves whether curves are smooth, as opposed to having corners.

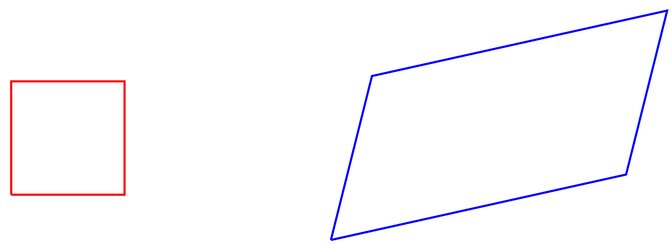
The above figures have different differential structures, since the right figure has two corners.

3.4 Affine structure

Involves whether continuous curves are *straight*, as opposed to curved. These two figures have the same differential but not affine structure:



Affine structure doesn't include distances (or angles). These have the same affine structure:



3.5 Metric structure

Distances and angles are now well-defined.

These kinds of structure come in a hierarchy, in the sense that higher levels, intuitively, build on lower levels. To get affine structure, for instance, you start with topological (or differentiable) structure and specify *more* structure: which continuous (or smooth) curves are to count as straight.

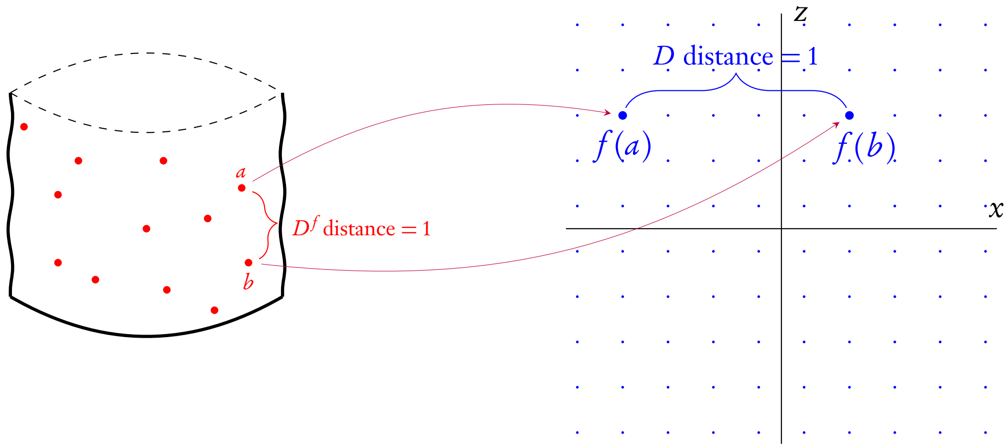
4. The metaphysics of spatial structure

Puzzle: I said that, e.g., distance isn't well-defined in the "bag of points". But there *are* functions from points in the bag to real numbers that formally behave like distance functions, i.e., which satisfy these constraints:

$$\begin{aligned} d(p,q) &\geq 0 \\ d(p,q) &= 0 \text{ iff } p = q \\ d(p,q) &= d(q,p) \\ d(p,q) + d(q,r) &\geq d(p,r) \qquad \text{(triangle inequality)} \end{aligned}$$

Let D be the distance function on some metric space with exactly as many points as the bag. Let f be a one-to-one function from points in the bag to points in the metric space. We can use f to pick out a "copy", in the bag, of the function D:

$$D^f(x,y) = D\big(f(x), f(y)\big) \qquad \text{(for any points } x \text{ and } y \text{ in the bag)}$$



Some metric space

Is the problem with D^f that many other functions satisfy the constraints for being a distance function? But even in the metric space, many functions other than the genuine metric D satisfy the constraints. (For any one-to-one function g from the metric space onto itself, we can construct a corresponding D^g.)

4.1 Metaphysically inflationary conception

D^f is disqualified because it is not *a natural kind*. D *is* a natural kind.

4.2 Coordinatization conception (Wallace, 2019)

A space has an associated set of *admissible coordinatizations* which together give the space its structure.

A coordinatization is a way of assigning ('tuples of) real numbers—"coordinates"—to each point in the space.

Inadmissible coordinatizations misrepresent the space's structure. (E.g., in a one-dimensional metric space, assigning to evenly spaced points p, q, r the coordinates $1, 2, 17$.)

Admissible coordinatizations are systematically related. In the one-dimensional metric space, where f is any admissible coordinatization, the set of admissible coordinatizations is the set of functions g such that:

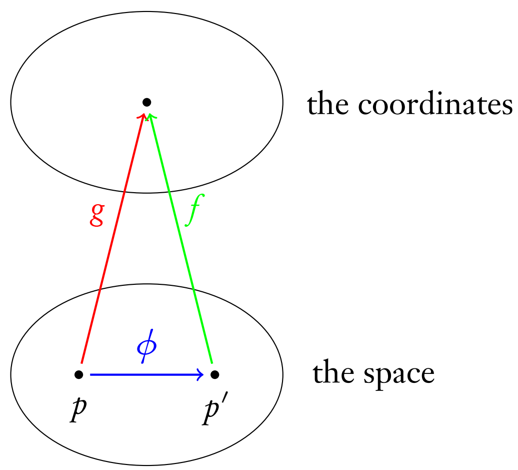
$$g(p) = af(p) + b$$

(for any point p, where a is either 1 or -1 and b is some real number)

The systematic relations will differ, depending on the structure of the space. E.g., for a one-dimensional merely topological space, different admissible coordinates can "stretch" and "contract".

Let S be a set of points and C a set of admissible coordinatizations of S.

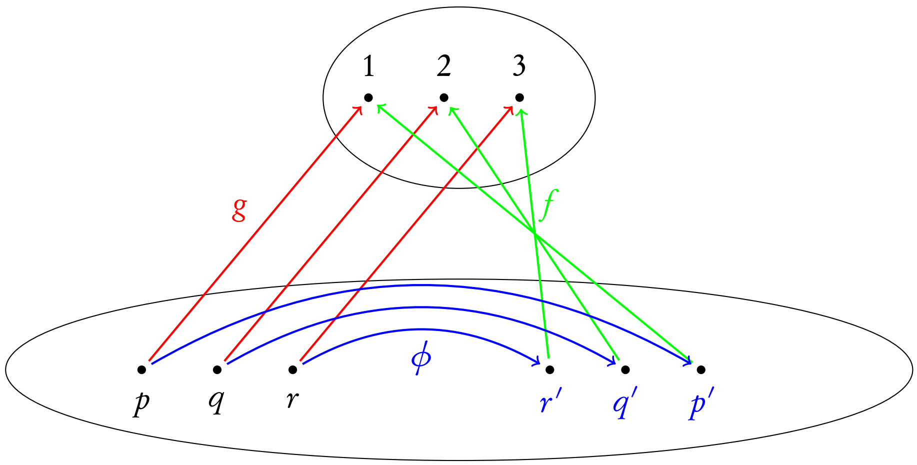
> An *automorphism* of $\langle S, C\rangle$ is a one-to-one function ϕ from S to itself of the form $\phi(p) = f^{-1}\big(g(p)\big)$, for any $f, g \in C$



> An n-place property P is intrinsic to $\langle S, C\rangle$ iff for every automorphism ϕ of $\langle S, C\rangle$ and any points $p_1, \ldots, p_n \in S$, $P(p_1, \ldots, p_n)$ iff $P\big(\phi(p_1), \ldots, \phi(p_n)\big)$

E.g., the the three-place relation *equally spaced* is intrinsic to the metric space because for any automorphism ϕ, points p, q, r are equally spaced iff the points

$\phi(p), \phi(q), \phi(r)$ are equally spaced. E.g.:



But the relation "left-of" is *not* intrinsic, since q is left of r but q' is not left of r'.

Back to the bag of points. D^f is not intrinsic because any one-to-one function whatsoever from the bag onto itself is an automorphism for a mere bag of points; and many properties derived from D^f won't be preserved by such automorphisms.

4.3 Which is better?

The metaphysical approach needs to make apparently arbitrary choices of what the natural kinds are (e.g., *continuous path* vs *noncontinuous path* vs *open set*).

But the coordinate approach has no satisfying answer to the question of why a function counts as an admissible coordinatization.

Also it isn't clear how the coordinate approach will characterize the question of whether absolute, rather than relative, distances are intrinsic. "Expansions" of coordinatizations (multiplying all coordinates by a constant factor) could be interpreted as either changing units, or doubling all distances.

References

Wallace, David (2019). "Who's Afraid of Coordinate Systems? An Essay on Representation of Spacetime Structure." *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 67: 125–136.