Math Logic
Homework #4 (Chapter 9)

Prove claim (9.15) on p. 109 of B&J. I want you to prove it rigorously, which means you must produce a proof by induction. However, some of the various cases in the proof may be extremely similar, in which case you don't need to write everything out — you can say "similar to the earlier case" if it's very obvious how the proof should go. Strong induction will probably be more useful than weak induction. You'll need to prove the equivalences (9.16), (9.17) and (9.18) along the way, and in doing so you'll probably find the following principles handy:

- (*) Let F be a sentence (formula without free variables) that doesn't contain the name *a*; let I be some interpretation; let I^a_o be the interpretation just like I except that it assigns object *o* to the name *a*. Then, $I^a_o(F)=I(F)$.

  Intuitively, (*) is true because the switch from I to I^a_o only affects the name *a*, and F was stipulated not to contain *a*. (Note that this is just what B&J later call "continuity").

- The principle of substitution of equivalents, principle (9.14) from B&J page 108.

For extra credit you could prove (*), but you don't need to; you can just assume it.