Math Logic
Homework #7 (Chapters 14 and 15)

1. Prove

 (*) For any (closed) term, t, in the language of arithmetic, there is a unique number, i, such that $\vdash_Q t=\mathbf{i}$

 Hint: use strong induction on the number of function symbols occurring in t.

2. Prove the following:

 Let ϕ be any quantifier-free sentence in the language of arithmetic that is true in the standard model. Show that ϕ is a theorem of Q.

 The lemmas proved in chapter 14 should be helpful, as well as (*) from problem # 1.

 Hint: it will be easier if you prove the following assertion (by strong induction on the number of connectives):

 Where ϕ is any quantifier-free sentence in the language of arithmetic containing only the boolean connectives ~ and &,

 i) if ϕ is true in the standard model then $\vdash_Q \phi$, AND
 ii) if ϕ is not true in the standard model then $\vdash_Q {\sim}\phi$

 The desired result follows from this, since every boolean connective can be defined in terms of ~ and &.

3. Show that the assignment of numbers to expressions given on p. 171 is indeed a gödel-numbering, in the sense defined on p. 170. To do this you'll need to describe a procedure for taking a given number and deciding whether it is the gödel-number of any expression, and if so, which expression that is. Describe this procedure in English (i.e., don't give me a Turing machine or anything like that -- just give the idea of how the procedure would go.)