Wallace on Math- vs. Language- First

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Seminar on fundamentality

1. Language-first versus Math-first

"Language-first": a scientific theory is a collection of *sentences*. E.g.:

> Every body perseveres in its state of being at rest or of moving uniformly straight forward except insofar as it is compelled to change its state by forces impressed… A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed… the common center of gravity of two or more bodies does not change its state whether of motion or of rest as a result of the actions of the bodies upon one another. (Wallace, 2022, p. 348)

"Math-first": a scientific theory is a collection of *mathematical models*. E.g.:

> A model of N-particle Newtonian mechanics is specified by:
>
> 1. A list of N positive real numbers $m_1, \ldots n_N$, representing the particle masses;
> 2. A list of $N(N-1)$ smooth potential functions $V_{nm} : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ representing the 2-particle potential between the pairs of particles and satisfying $V_{nm} = V_{mn}$;
> 3. A collection of N smooth functions $x_n : \mathbb{R} \to \mathbb{R}^3$ satisfying the differential equations
>
> $$m_n \frac{\mathrm{d}^2 x_n(t)}{\mathrm{d}t^2} = - \sum_{m=1, m \neq n}^{N} \nabla V_{nm}\big(|x_n - x_m|\big)$$
>
> (Wallace, 2022, p. 348)

The question is really about epistemology and representation, not ontology: what is the most perspicuous way to represent the "actual epistemic achievement" (Wallace, 2022, p. 349–50) of a scientific theory.

Some apparent differences between the approaches:

- The representational powers of sentences are "already there"; for models, they depend on scientist's context-dependent intentions.

  (But this is a matter of degree.)

- Sentences can be true or false; models are similar to the world, to varying degrees and along varying dimensions.

  (But credences in *similarity-propositions*, "Model M is similar to the world in respect R to degree D", play a central epistemic role, in confirmation and guidance. To be sure, they're *vague*.)

The legitimacy of vagueness—and unspecificity more generally—in scientific representation seems more important than math-first versus language-first.

2. Advantages of the Math-first approach

2.1 Approximation and domain restriction

Language-based theory: "Bodies in the solar system move in ellipses". But planets don't *exactly* move in ellipses; and not *all* bodies move even approximately like ellipses; and it's unclear how to refine the theory to fix this.

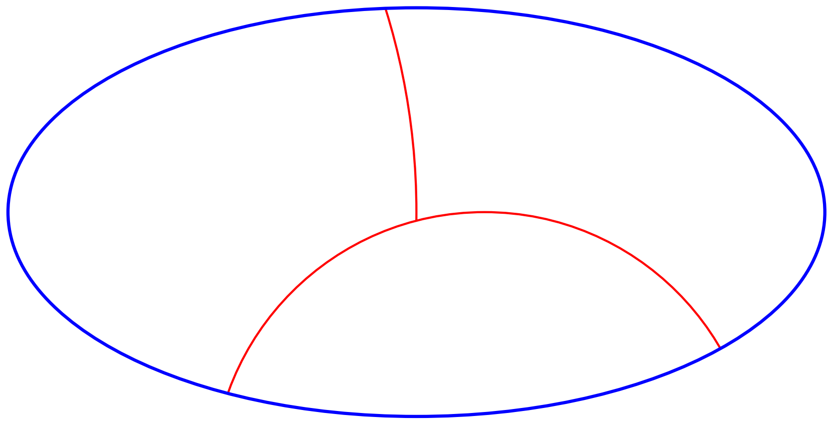
Wallace: things are smoother for the math-first approach since mathematical models are always understood as being good only at certain scales or domains.

Response: we could replace the language-based theory with the vague sentence "Many bodies in the solar system move approximately in ellipses", whose precisifications are parallel to the similarity propositions of the math-first view. Each view requires parallel vagueness when it comes to confirmation and guidance.

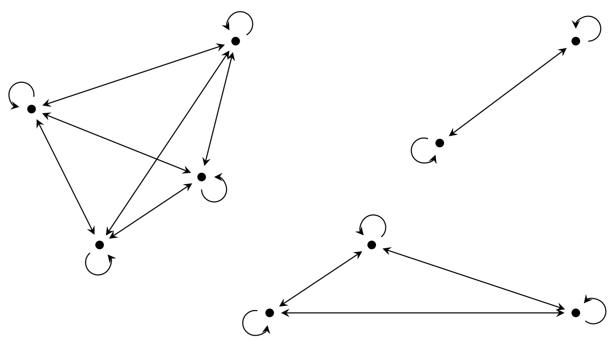
2.2 Equivalence

> On the math-first view, theoretical equivalence is something like equivalence by the standards of mathematics: a 1:1 transformation between models that preserves mathematical structure. Pinning that down precisely is no easier here than in the language-first context (set-theoretic isomorphism is too restrictive; categorical equivalence appears to be too permissive…). But it is relatively clear case-by-case, and a systematic feature of those cases is that theoretical equivalence is normally much more coarse-grained on the math-first than on the language-first view. (Wallace, 2022, p. 353)

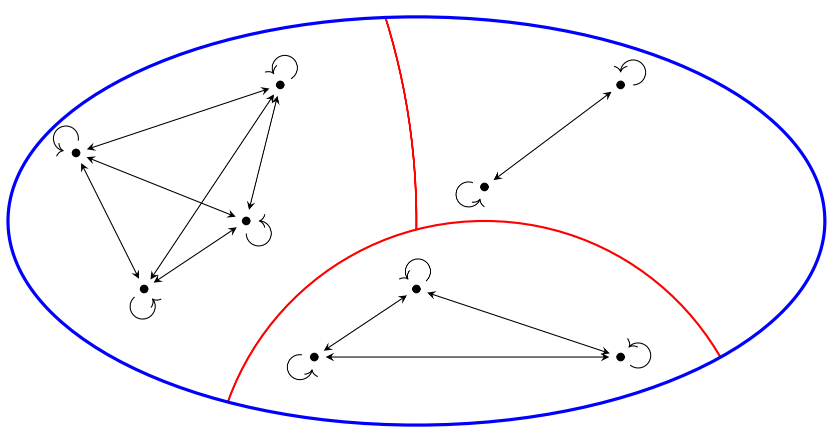
Example: A *partition* is a way of exhaustively dividing a set into nonoverlapping, nonempty subsets:



An *equivalence relation* is a reflexive, symmetric, and transitive relation:



You can go "back and forth" between these two concepts:



Why is theoretical equivalence coarser-grained on the math-first view?

> **Newtonian Particle Mechanics** A model of Newtonian N-particle mechanics is given by N smooth trajectories in Euclidean space. But that statement could be precisified as (inter alia):
>
> - N smooth (that is: infinitely-many-times differentiable) maps from the real line to $\mathscr{E}^3$, satisfying such-and-such differential equation.
> - N smooth curves (that is: dimension-1 submanifolds), in $\mathscr{E}^4 = \mathscr{E}^3 \times \mathscr{E}$, representing Newtonian spacetime.
>
> The former might naturally be translated into the language-first view via some function $\mathrm{Loc}(n,t)$, giving the location of particle n at time t; the latter by some 2-place predicate $\mathrm{Occupied}(x)$ that records the points of spacetime occupied by particles. Again, the prospects of intertranslatability look dim. (Wallace, 2022, p. 355)

The two corresponding language-first theories are:

> *Particles + space*: there exist N particles and a substantival three-dimensional (physical) space; particles are located at points of space at times.
>
> *Supersubstantival spacetime*: there exists only a four-dimensional substantival (physical) spacetime; some points in this space have a certain physical feature of being "occupied" ($\neq$ occupied-by-particles)

Regarding an earlier example, Wallace writes:

> The math-first view regards these as equally-legitimate ways of presenting the same theory, but any plausible attempt to throw the different descriptions into language-first form (say, by describing each in first-order logical language) will realistically fail to provide any purely-formal translation between those descriptions. (2022, p. 353)

Why? Because:

> On the language-first view, formal equivalence seems to be something like intertranslatability, or logical equivalence, or interdefinability. Equivalent theories are talking about the *same entities*, and saying the *same things* about them, just using different words or expressions. (2022, p. 353)

Particles + Space and Supersubstantival spacetime have different ontologies.

But the mere fact that the math-first approach uses mathematical models isn't what gives it its flexibility. It's also that it understands a theory as a *collection* of models. The attitude toward the collection seems to be one of "quotienting" (Sider, 2020, chapter 5). These aspects are separable.

Similarly, the language-first approach might adopt quotienting. Note that in the case of Theory 1 and Theory 2, this will require quantifier variance (which anyway seems assumed by Wallace's math-first approach).

References

Sider, Theodore (2020). *The Tools of Metaphysics and the Metaphysics of Science*. Oxford: Oxford University Press.

Wallace, David (2022). "Stating Structural Realism: Mathematics-First Approaches to Physics and Metaphysics." *Philosophical Perspectives* 36(1): 345–378.