Coordinates and Equivalence

Ted Sider
Seminar on fundamentality

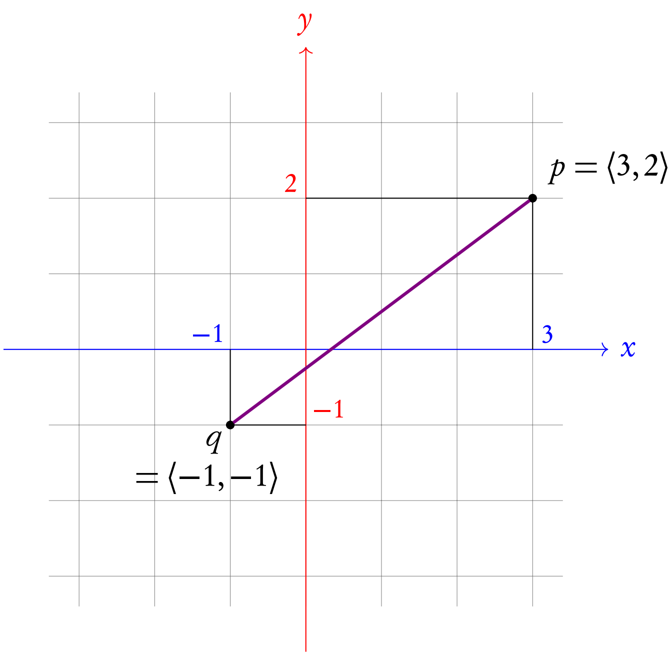
1. Intrinsic vs extrinsic formulations.

Assuming substantivalism, here are two ways to describe space's metric structure.

1.1 Coordinate-based approach

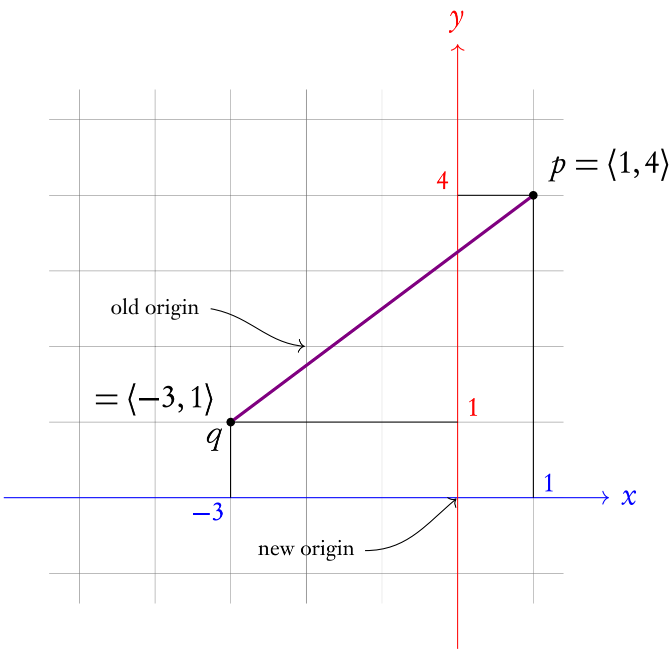
First, assign *coordinates* (certain mathematical objects) to each point in space.

Example: two-dimensional Euclidean space. Here the coordinates can be ordered pairs of numbers:



$$\begin{aligned} \mathrm{dist}(p,q) &= \sqrt{(\Delta x)^2 + (\Delta y)^2} \\ &= \sqrt{\left(3-(-1)\right)^2 + \left(2-(-1)\right)^2} \\ &= \sqrt{4^2 + 3^2} \\ &= 5 \end{aligned}$$

Second, note that any particular coordinatization is arbitrary. E.g., the origin can be shifted (that is, a different point can be assigned the coordinate $\langle 0,0\rangle$):



$$\begin{aligned}\mathrm{dist}(p,q) &= \sqrt{(\Delta x)^2 + (\Delta y)^2}\\ &= \sqrt{\big(1-(-3)\big)^2 + \big(4-1\big)^2}\\ &= \sqrt{4^2+3^2}\\ &= 5\end{aligned}$$

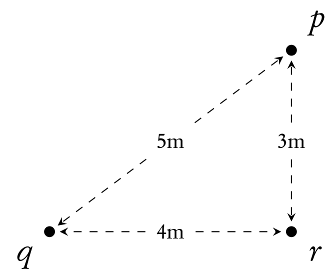
Other coordinate changes are possible, such as rotating or mirror-imaging the axes.

So: to describe the space, we specify the entire set of coordinate systems with which it can be represented.

1.2 Coordinate-free (or "intrinsic") approach

Here we describe the structure of physical space directly, without using coordinate systems:

> There is a set P of points of physical space
>
> That space is two-dimensional
>
> Between any two points in the space there is a certain distance in meters
>
> These distances are "Euclidean". For example, if three points, p, q, and r form a right triangle whose hypotenuse is segment pq, and if the distance between q and r is 4 meters and the distance between p and r is 3 meters, then the distance between p and q must be $\sqrt{4^2+3^2}$ meters, i.e., 5 meters:

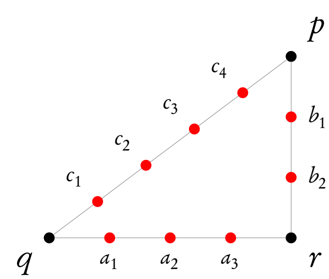


1.3 Differences between the approaches

1.4 Further insistence on intrinsicality

Field (1980) argues that the description of space should be even more intrinsic: we shouldn't use numbers at all. E.g., we can describe $\triangle pqr$ thus:

> Points p, q, and r, aren't co-linear.
>
> There exist "evenly spaced" points a_1, a_2, and a_3 between q and r, evenly spaced points b_1 and b_2 between p and r, and evenly spaced points c_1, c_2, c_3, and c_4 between q and p; and all these sequences are "equally spaced".



We can define all this using using just two primitive predicates, Bet x, y, z for *linear betweenness*, and x, y Cong z, w for *congruence*:

> "p, q, and r aren't co-linear": no two of these points are identical and none is between the other two (i.e., $\sim$Bet pqr, $\sim$Bet qpr, and $\sim$Bet prq.)
>
> "the a_is are evenly spaced and between q and r": no a_i is identical to either q and r; and Bet q, a_1, a_2 and Bet a_1, a_2, a_3 and Bet a_2, a_3, r; and q, a_1 Cong a_1, a_2 and a_1, a_2 Cong a_2, a_3 and a_2, a_3 Cong a_3, r. (Similarly for b_i and c_i.)
>
> "the sequences are equally spaced": q, a_1 Cong p, b_1 and q, a_1 Cong q, c_1

2. Structural realism

James Ladyman and others say that we should avoid bad metaphysical questions by understanding physical theories as not talking about entities, but instead talking about relational structures. But it's unclear what that means.

3. Equivalence

Some pairs of theories are *equivalent*, such as theories that differ only by units of measurement. But what does it mean to say that theories are equivalent?

3.1 Fundamentality/naturalness approach

Equivalent theories say the same thing about the natural properties and relations.

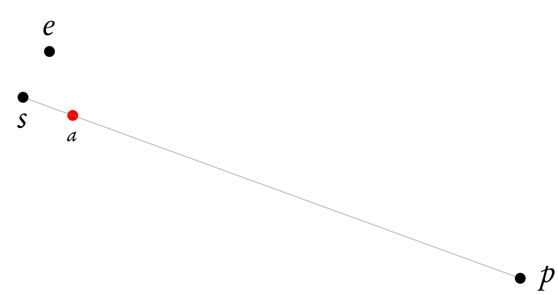
Illustration for theories differing only by the unit of measure for distance:

First, replace statements of distance using units, such as "The Earth is 96 million miles from the Sun", with comparative statements:

> The Earth is closer to the Sun than it is to Pluto
>
> The Earth is as close to Sun as Mercury is to Mars
>
> Mars is 1.5 times as far from the Sun as the Earth is

Second, define these comparative statements using Bet and Cong. Letting e, s, p, m and r be the centers of mass of the Earth, Sun, Pluto, Mercury, and Mars:

> "The Earth is closer to the Sun than it is to Pluto"
>
> ⇒ There exists some point a that is distinct from both s and p, such that Bet s, a, p and e, s Cong a, s.



‘The Earth is as close to the Sun as Mercury is to Mars”

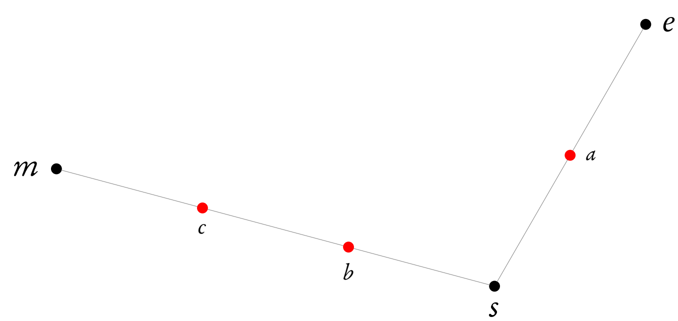
- $\Rightarrow$ $e, s \,\mathrm{Cong}\, m, r$

“Mars is 1.5 times as far from the Sun as the Earth is”

- $\Rightarrow$ There exists a point a (distinct from both s and e), and there exist points b and c (distinct from both s and m) such that:

  $\mathrm{Bet}\, s, a, e \qquad \mathrm{Bet}\, s, b, c \qquad \mathrm{Bet}\, b, c, m$

  $s, a \,\mathrm{Cong}\, a, e \qquad s, a \,\mathrm{Cong}\, s, b \qquad s, b \,\mathrm{Cong}\, b, c \qquad b, c \,\mathrm{Cong}\, c, m$



Third, assert that betweenness and congruence are perfectly natural relations.

Finally, claim that theories using different units for distance are equivalent because they describe the very same betweeness- and congruence- facts. (One can make this precise using representation theorems.)

3.2 Equivalence is “worldly”

Given the fundamentality approach, whether theories are equivalent can depend, not only on “internal” considerations, but also on the world, namely, on what the natural properties and relations are. For example, whether geometric conventionalists are right that “space is flat” and “space is curved” are equivalent descriptions depends on whether there are perfectly natural metrical spatial relations.

(But really, any scientific realist should agree that equivalence is worldly.)

3.3 The problem of difficult choices

In order to say that two theories about distance using different units are equivalent, the fundamentality approach requires us to find a *third* language, which uses more natural properties than the languages of the equivalent theories. But what if there is no such third language?

Example: disputes about ontology, such as:

Material composition Mereological Universalists (e.g. Lewis; composite objects like tables and chairs do exist—as do scattered objects) versus Mereological Nihilists (e.g., Dorr; no composite objects exist)

Ontology of space Substantivalists (e.g., Newton; points of space exist) versus relationalists (e.g., Leibniz; no they don't; only material bodies exist)

Quantum Bohmian ontology Low-dimensionalists (E.g., Maudlin; three-dimensional particles and/or points of space exist) versus high-dimensionalists (e.g., Albert; no they don't; only points in an extremely high-dimensional space, plus perhaps a single marvelous particle, exist)

Some (e.g., Hirsch, Wallace, and Wallace, respectively) say these debates are merely verbal; that apparently opposing views are in fact equivalent. But it's hard to see what the third language would be, since the relevant vocabulary—namely quantifiers—seems impossible to understand in other terms.

My view is that the fundamentality approach's verdict here is correct. These disputes are not merely verbal; the theories aren't equivalent.

However, aren't a pair of theories that differ only in that one uses $\forall$ and the other uses $\exists$ equivalent? But again, there seems to be no third language.

(The same point can be made with nonlogical cases, such as earlier-than vs later-than, or parthood vs overlap vs fusion.)

3.4 "Quotienting"

We can say <u>that</u> theories are equivalent without saying <u>why</u> they are equivalent in terms of fundamentality and underlying third theories.

Regarding $\forall$ vs $\exists$, a quotienter might say:

> A good theory can be formulated using the concept of $\forall$. But we can give an equally good theory using the concept of $\exists$. Indeed, I can give a precise account of a relation between theories that guarantees equivalence: the relation holding between theories when and only when they are identical save for an exchange of one or more occurrences of a quantifier Q with $\sim Q'\sim$. I don't have any more fundamental description of quantificational reality from which these theories can be viewed as getting at the same fundamental facts. But no such theory is needed; it's enough simply to say which theories are good ones and which ones are equivalent.

I think many metaphysicians tend to assume something like this:

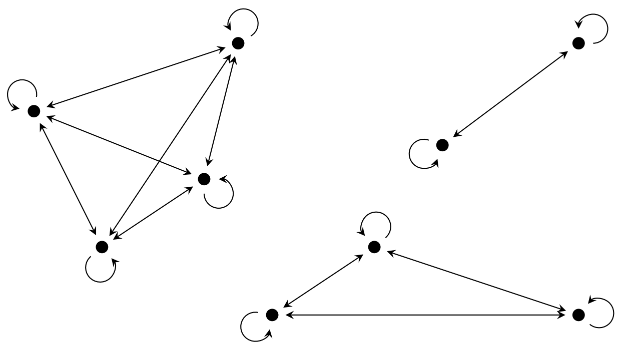
> It's ok to construct models of some phenomenon, with artifacts. But there must also be some way of describing the phenomenon that in some sense does not have artifacts, some way of saying what is really going on. An example of "saying what is really going on" is describing the distance facts using the relations of betweenness and congruence. This privileged description doesn't mention numbers at all (thus removing the artifact). And one can recover from this "privileged" description exactly which numerical models can be used to describe reality, and exactly which features of these numerical models are artifacts.

Quotienters reject this, and say instead:

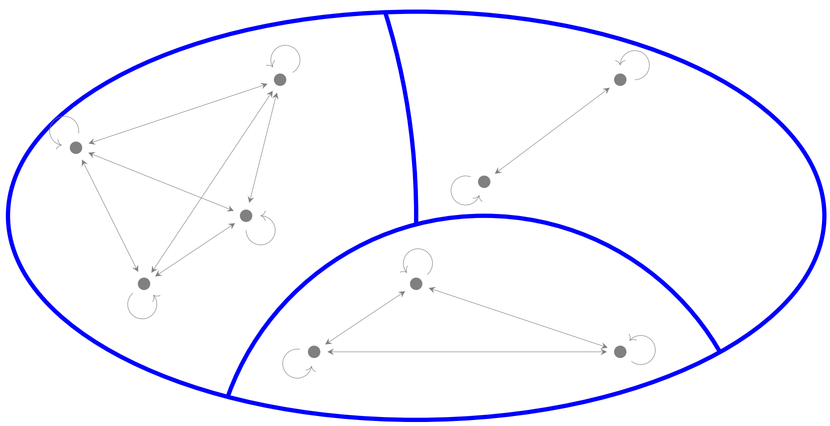
> There may be no way to say what is "really" going on; maybe every good model has artifacts. It's ok to just say: this model does a good job of representing the phenomenon, but certain features of the model are artifacts. Moreover, for any model, we can say which features of the model are genuinely representational and which are artifacts. There is no need to provide some privileged description that has no artifacts from which we can recover the information about models; we can just stop with the models.

I'm calling it "quotienting" because the quotienter thinks that, if we're given a set of theories with conventional differences, we can just "quotient out" the conventional content and regard the best description as an equivalence class of theories.

(This terminology comes from mathematics. Suppose you start with some objects, related by some reflexive, symmetric, and transitive relation:



If we don't care about the differences between the objects that stand in the relation, we can stop talking about the objects, and instead talk about their *equivalence classes*—i.e., sets of those objects within which every object bears the relation to every other:



The set of equivalence classes is called the "quotient set" of the original set of objects; shifting one's talk from the original objects to the equivalence classes is called "quotienting out" the differences between equivalent objects.)

I think that in the philosophy of physics, many people implicitly take a stand on quotienting, one way or the other, and that this often plays an important role in structuring what they regard as acceptable ways to theorize.

4. Against quotienting

The core issue is whether quotienters refrain from explaining something that ought to be explained. (This kind of disagreement about explanation is often at the core of disputes over the legitimacy of metaphysics.)

Imagine Leibniz saying merely "descriptions of objects in space are equivalent when and only when they differ only by some combination of global translations and rotations of the positions of material objects", and not going on to say: "such descriptions are equivalent *because* they agree on the fundamental facts, namely, the facts about distances between material bodies".

5. Defending the fundamentality/naturalness approach

Progress can be unexpected

Hard choices are hard to avoid If you accept some comparisons of naturalness (e.g., mass and charge vs grueified versions), it's hard to see how to avoid the legitimacy of comparisons like $\exists$ vs $\forall$ (or earlier-than vs later-than, etc.)

There can be more than one One could claim that *both* $\exists$ and $\forall$ are perfectly natural. The presumption in favor of fewer natural meanings, namely parsimony, is an epistemic consideration, and as such is defeasible. Perhaps other epistemic considerations, such as avoiding arbitrariness, outweigh it in this case.

Why think we can know everything? It's cliché but true that the "your proposed metaphysics leads to unknowable facts, so it should be rejected" argument is hard to defend in any principled way. Generalizations that eliminate metaphysical questions tend to also eliminate legitimate scientific questions.

Unknowability *can* be a sign that one is employing concepts that aren't in good standing. But if one's concepts play a central role in *other* questions that are part of legitimate inquiry, then that's a good reason to think that they are in good standing; and if they can then be used to raise a question that's hard to answer, well, that's life. We don't know everything.

5.1 Intermediate views?

Most of our instincts about equivalence, I suspect, are in the middle; both quotienting and the fundamentality/naturalness approach are too extreme. What might an intermediate view look like?

From the fundamentality/naturalness end: we could say that it's determinately the case that either ∀ or ∃ is vague, but that it isn't determine which one is vague (Dorr and Hawthorne 2013, pp. 63–5; Sud 2018).

> (Concern: assuming that the world itself isn't vague—that a "complete" description of it can be given in perfectly precise terms—and that expressions are vague when they have multiple precisifications, what would the precisifications of 'natural' look like?)

From the quotienting end: a quotienter might say that a mass+charge theory is inequivalent to a grueified version, but that the ∀ and ∃ theories *are* equivalent.

References

Dorr, Cian and John Hawthorne (2013). "Naturalness." In Karen Bennett and Dean W. Zimmerman (eds.), *Oxford Studies in Metaphysics*, volume 8, 3–77. Oxford: Oxford University Press.

Field, Hartry (1980). *Science Without Numbers*. Oxford: Blackwell.

Sud, Rohan (2018). "Vague Naturalness as Ersatz Metaphysical Vagueness." *Oxford Studies in Metaphysics* 11: 243–77.