Crash Course on Spatial Structure

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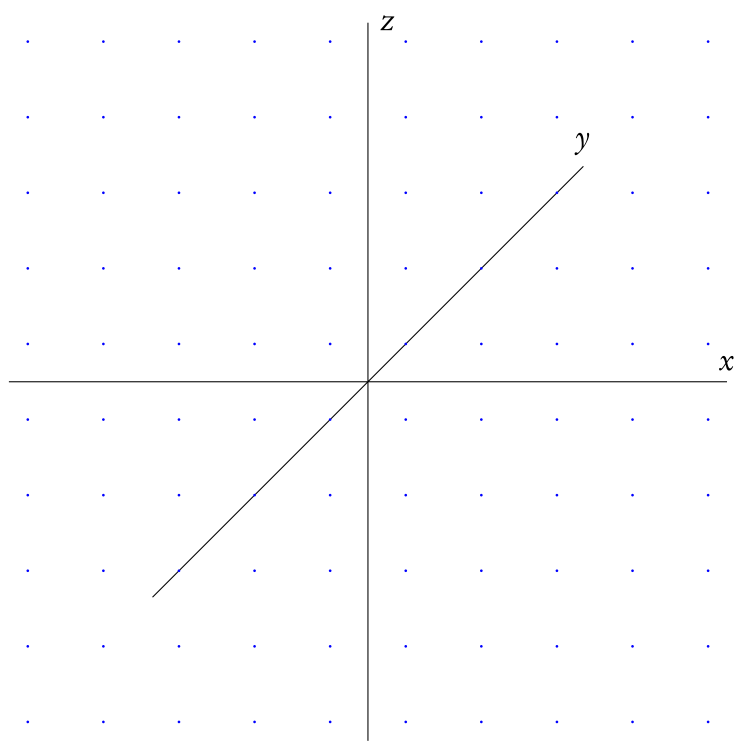
1. The idea of spatial structure

In the philosophy of physics, one of the central questions about space is: what is its "structure"?

For the purposes of our discussion, let's assume that "substantivalism" about space is true. That is, as Newton thought, there are these entities, points of space. They stand in various spatial relationships to one another, and are occupied by material bodies. (Material bodies only stand in spatial relations derivatively, by virtue of the spatial relations between the points they occupy.) The question of the structure of space is roughly what the spatial relationships between points are like. (According to a rival view, "relationalism", there are no points in space; there are only, as Leibniz thought, material bodies which stand in spatial relationships to one another; and the question of the structure of "space" is then instead that of what these spatial relationships between material bodies are like. Much of our discussion here will carry over to that point of view.)

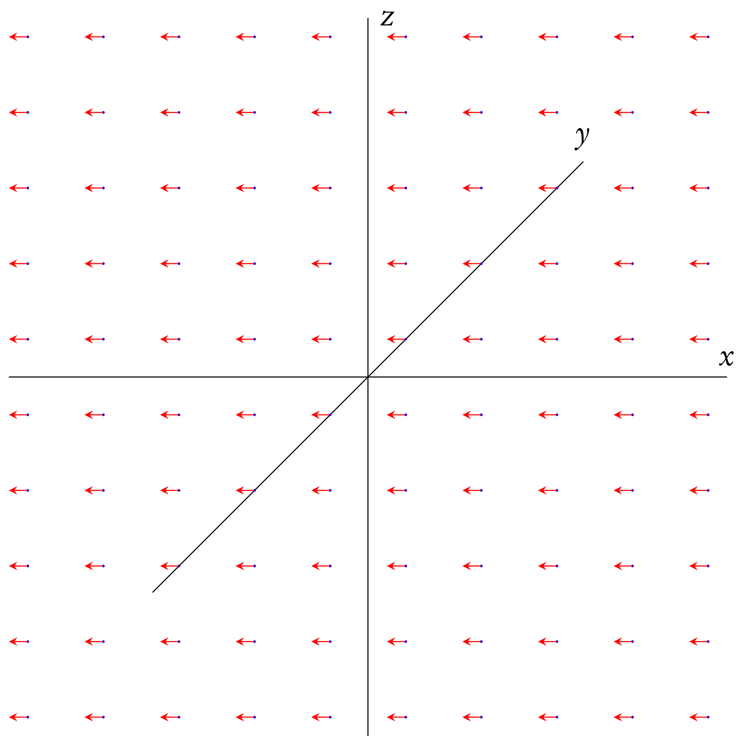
To introduce the idea of the "structure" of space, let's begin by talking about "left" and "right". Everyone knows that left and right are "relative". Which direction counts as left, or right, depends on which direction one is facing. There is no thing as "absolute" or "intrinsic" left and right, built into space itself. The notion of absolute left (as opposed to left relative to an orientation) is "not well-defined". It would be crazy to think that inherent in the nature of space itself is a single direction, "true left", so that there is a question of which direction is "truly left", irrespective of which direction one is facing.

Now, it isn't as if it is utterly *incoherent* to hold that space has a distinguished direction, call it L.[1] That is, it wouldn't be incoherent to claim that it is part of the inherent structure of space that "not all directions are created equally", but rather that there is a distinguished direction. On such a view, space itself shouldn't be pictured like this:



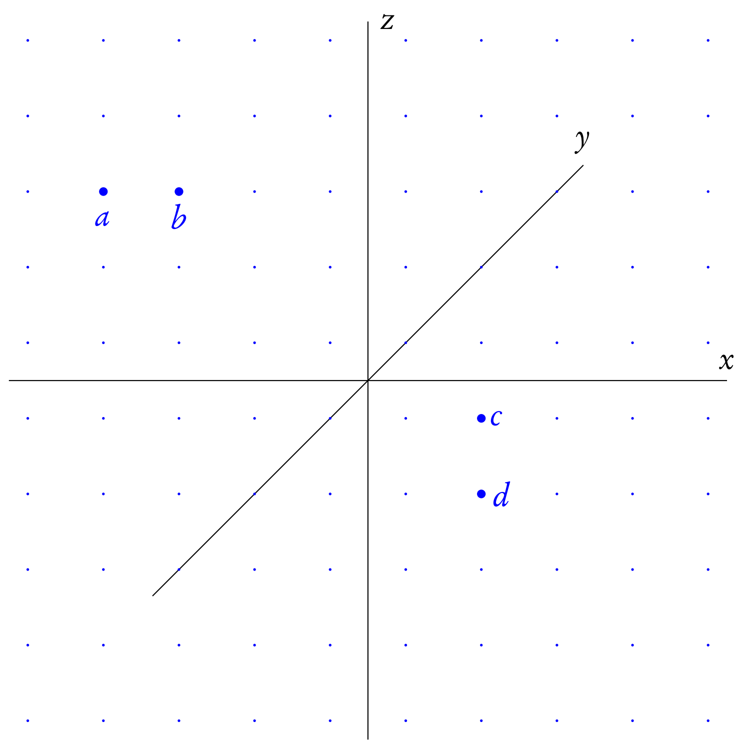
[1]Although it isn't incoherent to suppose space has a distinguished direction, maybe it *is* incoherent to suppose that such a direction could count as being *left*, given what we ordinarily mean by 'left'.

But rather like this:



(We'll talk more later about what it means, metaphysically, to say that space has one sort of structure as opposed to another; here I'm just introducing this idea intuitively.)

What might be an example of a feature of space that *is* built in? Well, the diagrams I've been using implicitly suggest, with their regular placement of points on the grid, and the three perpendicular coordinate axes, that there are built-in features of *distance*—that, for example, if you take four points of space, a, b, c, and d, there will be an objective, absolute, intrinsic-to-space fact of the matter whether a and b are *exactly as far apart* as are c and d:



It is a natural view that distances (or anyway, something like distances) are indeed built into space. However, this isn't uncontroversial; and it has been denied, for example by Reichenbach (1958, chapter 1).

So: the question of the structure of space is, in large part, the question of what spatial features are "intrinsic" or "absolute" or "built into space itself", as opposed to somehow being relative to facts about things in space (such as the orientation of an observer, in the case of left and right).

2. Laws and spatial structure

According to many, there is a connection between the question of which spatial features are absolute (intrinsic, inherent, etc.) and the laws of nature.[2] Suppose someone claimed that there was a law of nature saying that "all negatively

[2] See Earman (1989); Friedman (1983); Maudlin (2012); North (2021).

charged particles travel to the left". Of course, such a law is absurd. But it isn't absurd solely because it's empirically inadequate. It doesn't even make sense. Left and right are relative, after all; so how does a given negatively charged particle "know" which way to go? In order for a law to make sense, any geometrical notions it employs need to "make sense" in a nonrelative way.

What law of this sort *would* make sense, given that space is, in fact, isotropic—i.e., that it lacks a distinguished direction? For one, a law saying "all negatively charged particles travel in the same direction". Here's another. Suppose each electron was a spatially extended sphere, with a dot at one point that always moves parallel to the direction of motion of the center of mass of the electron. Then a law saying "all negatively charged particles turn 90 degrees left, with respect to the orientation defined by their current state of motion and their dot, every ten seconds" would make sense.[3]

For a more physically realistic example, consider Newton's first law of motion, which states that an object that isn't acted on by forces continues in its state of motion, whether at rest, or moving with uniform speed in a straight line. This law refers to two geometric notions: *being at rest*, and *moving in a straight line* (note that each involves time as well as space). Thus, according to the assumption we are discussing, Newton's first law could be a genuine law of nature only if being at rest, and moving in a straight line, are both built into the structure of space and time.[4]

3. Kinds of spatial structure

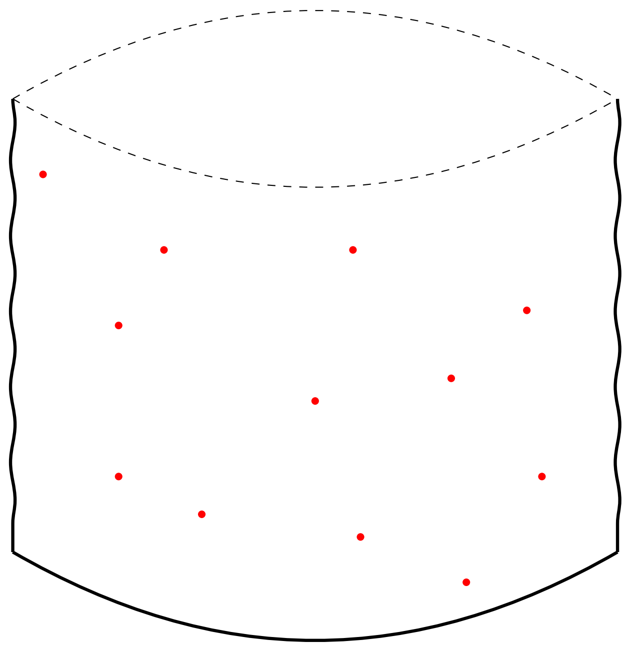
Next I want to talk about different kinds of spatial structure. These come in "levels", in a sense that will become clear. A full treatment of all of this would require a lot of mathematics, but the basic ideas can be conveyed informally.

The "lowest" or "simplest" level of structure is that given by the *number* of points: how many of them there are. Suppose a space had *just* this level of structure. It shouldn't then be pictured as in my diagrams above, since the points wouldn't have any particular spatial arrangement. Actually it's hard to

[3] Well, contrary to Kant, there may well be no distinguished notion of left-relative-to-an-orientation (that is, space may not be inherently "chiral"), in which case the stated law wouldn't make sense. However, the following *would* make sense: "every negatively charged particle turns, every ten seconds, in the same-handed direction as every other".

[4] See Maudlin (2012, chapter 1) for a full discussion this and related issues.

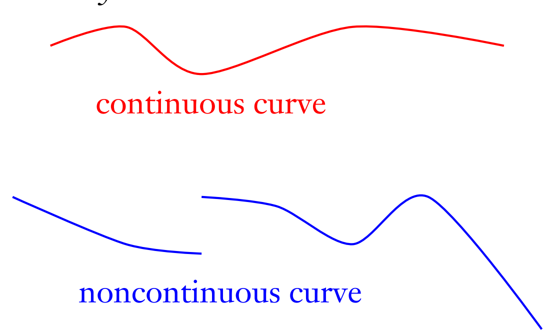
picture such a space at all, though perhaps we can do it like this:



A “bag of points”

Such a “space” isn’t really a space in any interesting at all, because it has no spatial structure. It’s a “mere set”; a mere “bag of points”. (Indeed, we might call this level of structure “set-theoretic”.)

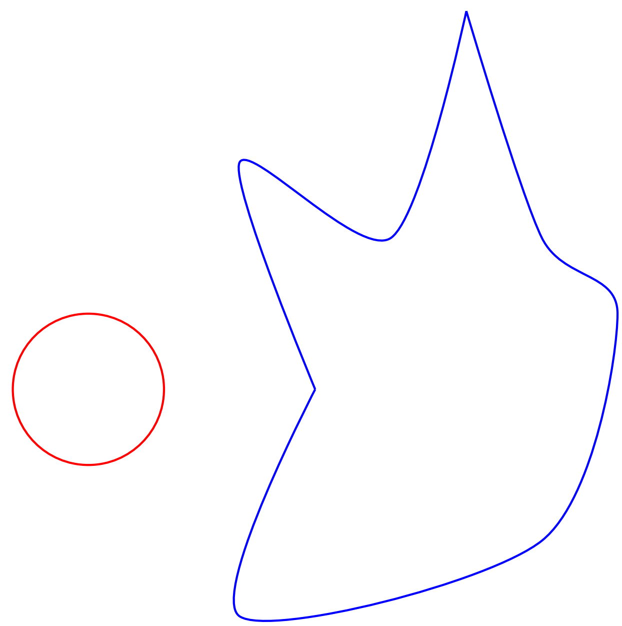
The next level of structure is “topological”. Topological structure has to do with notions like *continuity*:



The top curve is continuous because (roughly speaking) you could draw it without lifting your pen. The bottom isn’t.

Notice that continuity doesn't have anything to do with size or shape or straightness.

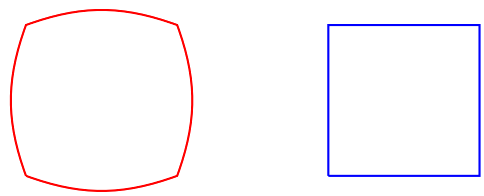
A space that is merely topological would have i) a set of points (and thus a definite number of points), and ii) a specification of which sets of those points count as continuous curves. (Actually the usual basic notion is *open set*.) But there wouldn't be any well-defined notion of what *shape* a given curve has. Whether it is "really" straight, whether it "really" has any sharp turns, how far apart its endpoints "really" are—all these simply wouldn't be well-defined questions. They would be like asking, of our actual physical space, whether a given direction is *really left*. To illustrate, these two figures are topologically the same:



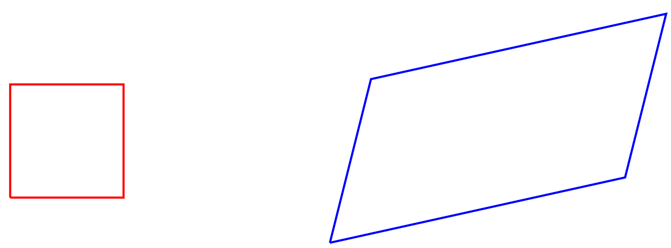
Each is a "closed" continuous curve. Of course, the figures don't look the same, but that's because we are visually sensitive to much more than topology: we perceive distances, angles, sharp points, etc.

The next level of structure is differentiable structure. Differentiable structure consists, not only of topological structure, but also of a specification of which curves are smooth, as opposed to having corners. In the preceding diagram, the two figures don't have the same differentiable structure, because the right hand figure has two "corners". But if those were smoothed out, then the figures would have the same differentiable structure—straightness and size aren't fixed by differentiable structure.

The next sort of structure, affine structure, specifies in addition which continuous curves are *straight*. These two figures have the same differential but not affine structure since only the right figure has straight lines:



But affine structure still doesn't include distances;[5] relatedly, it doesn't contain information about the angles between lines. These two figures have the same affine structure:



Finally we have *metric* structure, in which distances (and angles) are now built into the structure of the space. In the most familiar sort of space that has a metric, you can *move* a figure anywhere you like, and *rotate* it, without changing its intrinsic features. That's because such a space is homogeneous (no distinguished locations) and isotropic (no distinguished directions). But you can't resize a figure, let alone bend or straighten or smoothen it, etc.

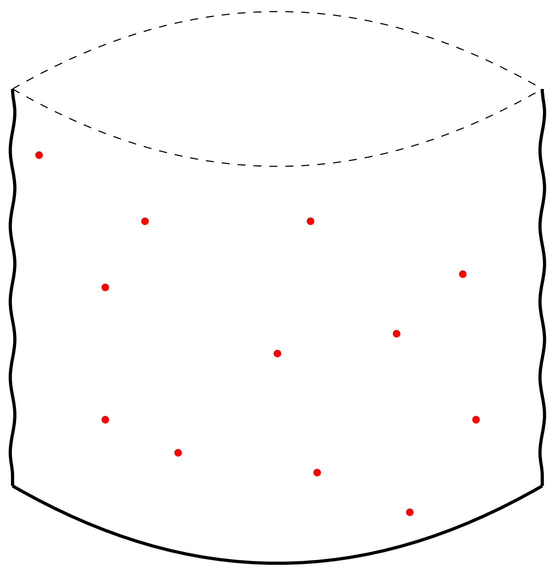
These various kinds of structure come in a hierarchy, in the sense that higher levels, intuitively, build on lower levels. To get affine structure, for instance, you start with topological (or differentiable) structure and specify *more* structure: which continuous (or smooth) curves are to count as straight.

[5] That's not quite right, actually; distances along parallel lines can be compared. That's why I rotated the parallelogram in the next figure.

4. The metaphysics of spatial structure

We have been speaking of structure that is "built into" space. But what does that mean, metaphysically speaking? This is an interesting and difficult question.

To appreciate why it's difficult, recall the bag of points:



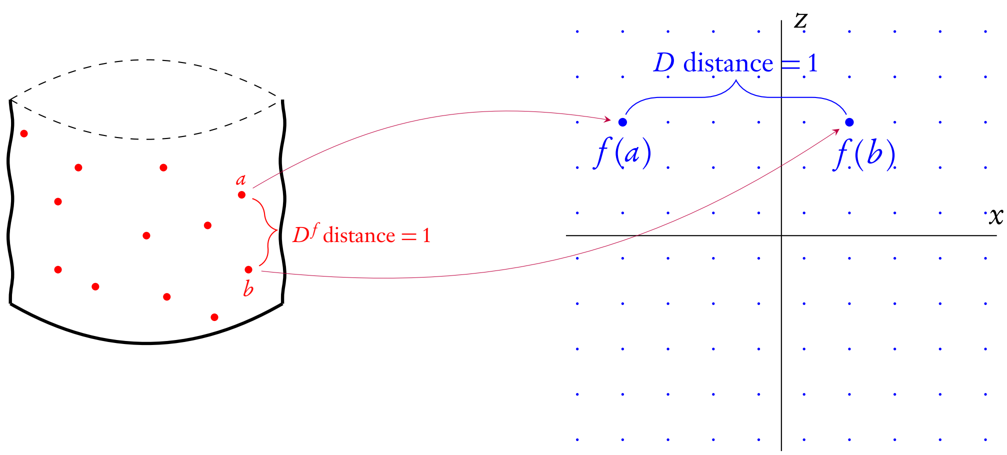
In the bag, I said, there are no facts about continuity, straightness, distance, etc.; these notions are not "well-defined". However, there *are* functions from pairs of points in the topological space to real numbers that formally behave like distance functions, i.e., which satisfy these constraints:

$$d(p,q) \geq 0$$
$$d(p,q) = 0 \text{ iff } p = q$$
$$d(p,q) = d(q,p)$$
$$d(p,q) + d(q,r) \geq d(p,r) \qquad \text{(triangle inequality)}$$

For consider some metric space that has exactly as many points as are in the bag; let D be its distance function. Since the metric space has the same number of points as the bag, there exists some one-to-one function, f, from points in the bag to points in the metric space. (This is the standard definition of two sets having the same number of members.) We can then use f to pick out a "copy", in the bag, of the function D. Since D satisfies the constraints, the copy function will be guaranteed to satisfy the constraints as well. The copy function, D^f, can be defined as follows:

$$D^f(x,y) = D\big(f(x), f(y)\big) \qquad \text{(for any points } x \text{ and } y \text{ in the bag)}$$

For instance, in the following diagram, the "copy distance" (i.e., distance in the sense of D^f) between points a and b in the bag is 1, because that is D-distance between the points $f(a)$ and $f(b)$ in the metric space to which a and b are mapped by f:



Some metric space

What one *wants* to say is that the function D^f isn't "built into" the structure of the bag of points, but rather needed to be imposed from the outside. But although we used the metric space to *pick out* D^f, it's a perfectly good function (mapping from pairs of points in the bag to numbers), and we don't need to think of it as "making reference to" the metric space. We could just think of this function as a set of ordered pairs which are composed of members of the bag and numbers: $\Big\{\big\langle\langle a,b\rangle,1\big\rangle,\ldots\Big\}$.

One might try saying that D^f doesn't really establish facts about distance in the bag because of the fact that there are many other functions from pairs of points in the bag to numbers that formally behave like distance functions. For example, if we picked any other one-to-one function f' from the bag onto the metric space, we could construct a different corresponding function $D^{f'}$. But although this thought is somewhat on the right track, it won't do as stated. For even in the metric space, which really does have "built-in distance facts", there are many functions other than D that formally behave like distance functions. For any one-to-one function g from the metric space onto the metric space, we can construct a corresponding function D^g that also formally behaves like a distance function (if D satisfies the four constraints then so will D^g). These other functions D^g clearly are *not* the "built-in" distance functions. (They will, in general, differ greatly from D in what they say—and not just by a change in unit of measure. They will treat different pairs of points as being congruent to one another, they will assign different ratios of distances, etc.) So the mere fact that a function with the right formal properties, such as D^f, is accompanied by other such functions isn't what disqualifies that function from being a "built-in distance function".

So what *does* disqualify D^f?

4.1 Metaphysically inflationary conception

Here is a metaphysically inflationary answer. (I say this without prejudice; I myself think this answer is correct.) D^f is disqualified because it is not *a natural kind*. D, on the other hand, *is* a natural kind.

This answer rests on a realist metaphysics of natural kinds. According to this metaphysics, properties, relations, etc., can be divided into two groups: the natural kinds, like green, blue, and D, say, on one hand, and the non-natural kinds, like Goodman's (1955) "grue" and "bleen", and D^f on the other.

(A lot more could be said, both about realism about natural kinds in general, and also about its application to the present case.[6] Because D involves numbers, I would actually prefer not to regard D itself as a natural kind, but would rather regard as natural kinds certain properties and relations that would allow us to prove representation theorems that establish the existence and uniqueness of functions like D.[7])

There is a sense in which realism about natural kinds is playing the role here of a label on a problem rather than a solution. Suppose one is a primitivist about

[6] For a quick introduction, see Sider (2024); for a fuller survey see Dorr (2019).

[7] See Sider (2011*a*) for an introduction to these issues.

natural kinds, in the sense that one isn't giving any further definition of what the natural kinds are. Calling D a natural kind and D^f not a natural kind is then close to just saying that D is part of the built-in structure of the metric space and D^f isn't part of the built-in structure of the bag. Put another way, the answer "D^f isn't a natural kind" shouldn't be seen as more of an intellectual advance on "D^f isn't built-in" than it really is. In particular, we aren't making sense of the idea of "built-in" in wholly other terms. The intellectual advance, such as it is, lies elsewhere: it is embedding the notion of natural kind into an overarching and hopefully explanatory theory.

4.2 Coordinatization conception

Here is a very different way to think about spatial structure: the built-in features of a space are those that are invariant amongst the space's *admissible coordinatizations*.[8]

To explain this approach, let's begin by explaining what "coordinatizations" are. A coordinatization of a space is simply an assignment of certain mathematical objects to points in the space—that is, a function, f, that maps each point p in the space to a mathematical object $f(p)$, which we think of as p's coordinate under f. Cartesian coordinates are a familiar example: a Cartesian coordinatization of a two-dimensional space is a function that assigns an ordered pair $\langle x,y\rangle$ of real numbers to each point in the space.

(It's important to distinguish between the space and the coordinates. The "spaces" we are talking about here are collections of physical points, whereas the coordinates are mathematical objects. Confusingly, it's common to also refer to the set of coordinates—for instance, the set of all ordered pairs of real numbers—as being a "space". We can disambiguate by calling sets of coordinates "mathematical spaces", and sets of physical points "physical spaces". By 'space' here I mean physical space.)

Coordinatizations are not unique; a given space will have many coordinatizations. Returning to the example of Cartesian coordinates: it is arbitrary where we put the origin. That is, it is arbitrary which point in the space is assigned the coordinate $\langle 0,0\rangle$. It's also arbitrary which directions in the space the x and y axes point in. That is: in any coordinatization, the x-axis is the set of points with

[8]See Wallace (2019, 2022). This approach—and perhaps also certain intermediate approaches—are common within philosophy of physics.

coordinates of the form $\langle x, 0\rangle$ (x varies over all the real numbers); the y-axis is the set of points with coordinates of the form $\langle 0, y\rangle$ (y varies); and it's arbitrary in which direction these axes point. Thus there are different coordinatizations that make these arbitrary choices differently.

The next notion to be introduced is that of an *admissible* coordinatization. An admissible coordinatization is a coordinatization that, intuitively, adequately reflects the intrinsic structure of the space. For example, consider a one-dimensional metric space. For such a space, the coordinates will be individual real numbers. However, not every function that assigns a real number to each point in the space will appropriately capture the intrinsic facts about the space. For instance, where p, q, and r are three "equally spaced" points (q is halfway between p and r), a coordinatization that assigns the coordinates 1, 2, and 17 to p, q, and r wouldn't accurately reflect the distance facts—it would represent p and q as being closer together than q and r—and so we call such a coordinatization *inadmissible*. To count as admissible (for this one-dimensional metric space), a coordinatization must assign "equally spaced real numbers" to p, q, and r, such as 1, 2, and 3, or 997, 998, and 999.

We can now state the approach:

> Any space is associated with a set of admissible coordinatizations; and these admissible coordinatizations, taken together as a group, characterizes the intrinsic structure of the space.

What does "associated with" mean? What determines what the range of admissible coordinatizations is? We'll talk about this question more later, but here is the idea: we *don't* give some further, "metaphysically deeper", account of what the intrinsic structure of the space is, from which we can derive the set of admissible coordinatizations. Instead, we say simply that the intrinsic nature of the space determines what its admissible coordinatizations are, and leave it at that.[9] And in order to describe the intrinsic structure of a space, we say what its admissible coordinatizations are, and leave it at that. (The claim is not that the set of admissible coordinatizations is somehow metaphysically basic. The spirit of this second approach is to reject the need for giving a metaphysically basic account of spatial structure—and indeed, perhaps to reject talk of metaphysical

[9] A further wrinkle is that the intrinsic nature of the space only selects what the admissible coordinatizations are once we have made certain conventional decisions of representation, such as what kinds of mathematical entities we are using to represent points.

basicness altogether.)

Let's illustrate this approach using the example of a one-dimensional metric space. The structure of this space is given by the set of its admissible coordinatizations. That is, for any intrinsic feature of the space, it is possible to "read off" that feature by inspecting the totality of admissible coordinatizations. A simple example of this is the space's dimensionality: the fact that the space is one-dimensional can be read off of the fact that each admissible coordinatization is a function that assigns a single real number (rather than, say, an ordered pair or an ordered triple of real numbers) to each point. Another example is the fact that our points p, q, and r are equally spaced: this can be read off of the fact that each admissible coordinatization assigns to p, q, and r three "evenly spaced real numbers" (that is, numbers, x, y, and z, with y between x and z, such that $|x-y|=|y-z|$).

Both *similarities* and *differences* between admissible coordinatizations play a role in giving the intrinsic features of a space. To illustrate, let's continue with the one-dimensional space. As I claimed in the previous paragraph, each admissible coordinatization assigns evenly-spaced real numbers to p, q, and r. This is a *similarity* between admissible coordinatizations, and it corresponds to the fact that the spatial even-spacing of p, q, and r *is* an intrinsic feature of the space. More generally, any two coordinatizations are similar in that they assign the same numerical distances to all pairs of points (that is, the absolute value of the difference between the coordinates of any pair of points is the same); this corresponds to the fact that (physical) distances between points *is* an intrinsic feature of the space. On the other hand, different coordinatizations will assign different numbers to these points—one assigns the number 0 to p, another assigns 997 to p, and so on. This is a *difference* between coordinatizations, and it corresponds to the fact that a "distinguished location" is *not* part of the intrinsic feature of the space. That is, there is no "distinguished origin"—different coordinatizations assign the coordinate 0 to different points. Another example of a difference between coordinatizations is this: whereas one coordinatization assigns increasing values to the points p, q, and r, namely 1, 2, and 3, other coordinatizations assign decreasing values to those points (albeit always equally-spaced coordinates), such as 3, 2, and 1, or 500, 499, and 498. This corresponds to the fact that a "distinguished direction" is *not* part of the structure of the space; the space is isotropic. These similarities and differences can be summarized as follows. Let f be any admissible coordinatization; then, any function g from points to real numbers is admissible if and only if it is just like f except that i) the origin may have moved, and ii) the points may be mirror-imaged—that is, if and only if there is some real number, b, such that g is related to f by one of the following equations:

$$g(p) = f(p) + b \qquad \text{(for any point } p\text{)}$$
$$g(p) = -f(p) + b \qquad \text{(for any point } p\text{)}$$

(b corresponds to a shift in coordinates, and whether the second or first equation relates g to f corresponds to whether g is a reflection of f.) This summary corresponds to a very general fact about the intrinsic structure of the one-dimensional metric space, namely that—to put it roughly—the "built-in" features of the space are *exactly* the totality of distances between points. For the functions g related to f by one of the two equations above are exactly those functions that assign the very same inter-point distances as f.

Let's take another example: consider a one-dimensional space that has merely topological structure.[10] The coordinatizations again assign individual real numbers to points. Suppose, as before, we have a coordinatization that assigns the numbers $1,2,3$ as the coordinates to three points p,q,r. Earlier, the fact that $1,2,3$ were evenly spaced numbers was representationally significant: it reflected the fact that the points p,q,r were evenly spaced. But here in the topological space, "evenly spaced" isn't a well-defined notion. How is this accounted for, in the present approach? By the fact that there are *also* other coordinatizations in which the coordinates assigned to p,q,r are *not* evenly spaced. E.g., there will be an admissible coordinatization in which the coordinates are $1,2,57$. However, there won't be any admissible coordinatizations in which the coordinates of p,q,r are $7,2,10$. For the first coordinatization represents q as being between p and r; and *betweeness* is indeed a topologically significant notion (in a one-dimensional space); so every admissible coordinatization must represent q as being between p and r. In general, you can move from one coordinatization to another by "stretching" coordinates, but you can't change the order or introduce gaps in coordinates.

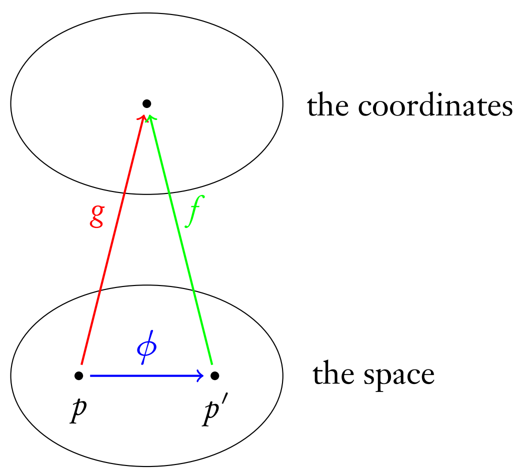
Next let's show how to give a precise definition of the locution 'property P is intrinsic to space $\langle S,C\rangle$', where S is a set of points and C a set of admissible

[10]Dimensionality, in fact, is indeed a topologically well-defined feature. That is, it's possible—though highly nontrivial—to give a definition, using only topological notions, of how many dimensions a set of points has.

coordinatizations.[11] First define the notion of an automorphism:

> An *automorphism* of $\langle S, C\rangle$ is a one-to-one function ϕ from S to itself such that for some $f, g \in C$, $\phi(p) = f^{-1}\big(g(p)\big)$ for each $p \in S$.

In other words, to define an automorphism, you pick any two coordinatizations f and g. Then if you begin with any point in the space, take its g coordinate—which is a number or 'tuple of numbers—and find the point in the space that has that number (or tuple) as its f coordinate, the second point is what the automorphism maps the first point to:



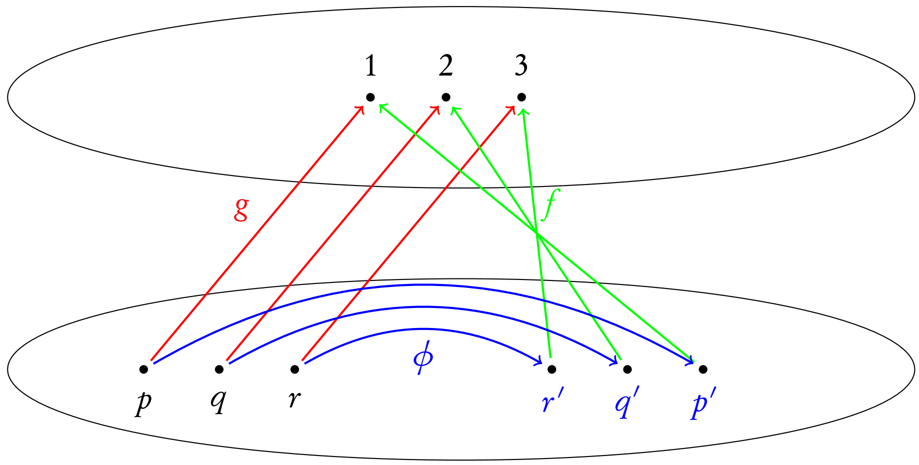
We can then say:

> An n-place property P is intrinsic to $\langle S, C\rangle$ iff for every automorphism ϕ of $\langle S, C\rangle$ and any points $p_1, \ldots, p_n \in S$, $P(p_1, \ldots, p_n)$ iff $P\big(\phi(p_1), \ldots, \phi(p_n)\big)$

For example, return to the one-dimensional metric space, and take the three-place relation of being *equally spaced*. To say that this property is intrinsic to the metric space is to say that for any automorphism ϕ, three points p, q, r are equally spaced iff the points $\phi(p), \phi(q), \phi(r)$ are equally spaced. And this will indeed be true, given what we saw earlier: that the admissible coordinatizations

[11] What follows works only if all the coordinatizations are total; would be worth reworking to make this assumption unnecessary.

of that space differ only by reflecting or shifting coordinates. For example:



In coordinatization g, the points p, q, r are assigned as coordinates the numbers $1, 2, 3$. Coordinatization f is a mirror reflection (and perhaps shift) of g, in which the coordinates of some other points p', q', r' are also $1, 2, 3$. These other points will be mirror imaged and shifted from p, q, r, but they too will be evenly spaced.

On the other hand, consider the relation "left-of" on this same metric space, by which I mean the relation of being being to the left of in the diagram, or, better, the relation that holds between points x and y iff the x-to-y direction is the same as the p-to-q direction. This relation is *not* intrinsic to the space, since, e.g., q is left of r but $\phi(q)$ (i.e., q') is not left of $\phi(r)$ (i.e., r').

Finally, let's see how the coordinate approach will understand the claim that the bogus "distance function" D^f on the bag of points considered earlier (which was derived via an arbitrary one-to-one function from the bag of points onto some metric space) isn't intrinsic to the bag of points. A mere bag of points, I said, has "no structure at all", beyond the mere facts of identity and difference between the points. On the present approach to structure, this can be understood as the following claim about the set of admissible coordinatizations of the bag. Choose C to be any set of mathematical entities with the same cardinality as the bag, to serve as the coordinates; then *every* one-to-one function from the bag onto C is an admissible coordinatization. It follows that any one-to-one function from the bag onto itself counts as an automorphism of the bag. Thus it follows, for instance, that relations like "D^f-congruence", and "D^f distance 1" (i.e., the four place relation holding between p, q, r, s such that $D^f(p,q) = D^f(r,x)$, and the two-place relation holding between p and q iff $D^f(p,q) = 1$) are not intrinsic to the bag, since it will be easy to find automorphisms that don't preserve them.

4.3 Which is better?

The coordinate-based approach has a certain advantage over the metaphysically inflationary approach. Take a space with merely topological structure. The inflationary approach will need to identify the natural kinds that are responsible for a given sort of structure. If a space has intrinsic topological structure, for instance, this must be because some particular property or relation is a natural kind. Perhaps it is the property of *being a continuous path*. But hang on. Another property that would suffice would be the property of *being an open set*. And for that matter, so would the property of *not being a continuous path*. We face a question: which of these is the natural kind? Is it open set? Is it continuity? Or perhaps both are natural kinds? Or perhaps neither are natural kinds (the natural kind being something else—noncontinuity, say)?[12]

These are puzzling questions that simply don't arise under the coordinate-based approach. Identifying the class of admissible coordinate functions for the topological space doesn't require privileging any of the above properties over the others as being a natural kind; all of them are preserved under any of the associated automorphisms.[13]

This advantage, though, is the flip side of a disadvantage. If we ask *why* this particular class of functions is the admissible coordinatizations, the defender of the second approach can give no answer. Whereas the defender of the metaphysical approach can say something that is intuitively more satisfying: these functions preserve topological structure because they preserve what the open sets are; and the open sets are part of the nature of space.

(The dialectical situation here is directly parallel to the opposition I discuss in *The tools of metaphysics and the metaphysics of science*, chapter 5, between a metaphysical approach to theoretical equivalence and an approach I call "quotienting".)

[12] See Sider (2011*b*, section 10.2) and Sider (2020, chapter 5) on this issue.

[13] Another advantage of the coordinate-based approach is that it isn't clear whether an adequate natural-kinds approach to differential structure is possible. The best attempt in this vicinity is Arntzenius and Dorr (2011).

Actually I have a second concern about the coordinate approach. How, under that approach, will we understand the question of whether "absolutism" about distance is true—that is, whether absolute distances (such as being one meter apart) are intrinsic to a space, or whether instead "comparativism" is true, according to which only ratios of distances are intrinsic?

A natural thought is that this is a question of whether "expansions" of coordinatizations are allowed. Return to our one-dimensional metric space. We said earlier that if we start with a coordinatization, f, we can get new coordinatizations by mirror-imaging and shifting the origin. But perhaps we should also be allowed to "expand" coordinatizations, by multiplying every coordinate by some fixed real number. If we do allow that, admissible coordinatizations needn't assign the same numerical differences to pairs of points; they need only assign the same ratios to any two pairs of points. It might be said the absolutism/comparativism question is the question of whether expansions of admissible coordinatizations are themselves admissible coordinatizations; absolutists say no whereas comparativists say yes.

But even an absolutist thinks that the numerical scale is arbitrary. This arbitrariness will need to be expressed in some way other than saying that a range of reprsentations, differing over what scales they represent, are all admissible. But the arbitrariness in where the origin is seems akin to the arbitrariness in what distance is represented by numerical distances of 1.

The metaphysical approach, in contrast, has an easy time with this issue. According to the comparativist, the natural kinds are relations like spatial congruence (the distance between p and q is the same as the distance between r and s), whereas according to the absolutist (simple case) the natural kinds also include relations like *being five meters apart*.

I think the coordinativist should say that there is an ambiguity in the use of coordinates. On one hand they could be used to represent different scales, so that (in the case of a metric space) the differences between coordinates aren't significant. On the other hand, the differences might not be taken to represent differences in scales. In the latter case, the differences are significant, if absolutism is true. We must use the latter understanding in order to characterize the absolutism/comparativism debate.

The need for a stipulation here might be defended by observing (as Wallace does) that *all* the use of real numbers to represent physical structure is conventional/stipulative. What's the harm if we need to make further stipulations about what we are using numbers to represent?

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