Axiomatic system K:

- *Rules*: MP plus necessitation:

$$\frac{\phi}{\Box\phi}\ \text{NEC}$$

- *Axioms*: all instances (with MPL-wffs) of PL1–PL3, plus:

$$\Box(\phi\rightarrow\psi)\rightarrow(\Box\phi\rightarrow\Box\psi) \qquad \text{(K)}$$

Some K-theorems

$$\Box(\phi\rightarrow\psi)\rightarrow(\Diamond\phi\rightarrow\Diamond\psi) \qquad \text{(K}\Diamond\text{)}$$

$$\vdash_{\text{K}} \sim\Box\phi\rightarrow\Diamond\sim\phi \qquad \vdash_{\text{K}} \Diamond\sim\phi\rightarrow\sim\Box\phi \qquad \text{(MN)}$$
$$\vdash_{\text{K}} \sim\Diamond\phi\rightarrow\Box\sim\phi \qquad \vdash_{\text{K}} \Box\sim\phi\rightarrow\sim\Diamond\phi$$

K-strategies

- **Proving $\Box\phi$**

  (Prove ϕ, then use NEC)

- **Proving $\Box\phi\rightarrow\Box\psi$**

  (Prove $\phi\rightarrow\psi$, use NEC, distribute the $\Box$ using a K axiom and MP)

- **Proving $\Box\phi\rightarrow(\Box\psi\rightarrow\Box\chi)$**

  (Prove $\phi\rightarrow(\psi\rightarrow\chi)$, use NEC, distribute the $\Box$ twice using a K axiom and MP)

- **Proving $\Box\phi\rightarrow(\Diamond\psi\rightarrow\Diamond\chi)$**

  (First prove $\phi\rightarrow(\psi\rightarrow\chi)$, then use NEC, then distribute the $\Box$ using a K axiom and MP, then use K$\Diamond$ and MP to distribute and transform the second $\Box$ to a $\Diamond$)

- **Use PL as needed**

  (E.g.., to move from the output of one of these strategies to what you need, or to move from what you have to the input of one of the strategies)

Axiomatic system D:

- Rules: MP, NEC
- Axioms: the PL1-, PL2-, PL3-, and K-schemas, plus the D-schema:

$$\Box\phi\rightarrow\Diamond\phi \qquad \text{(D)}$$

Axiomatic system T:

- Rules: MP, NEC
- Axioms: the PL1-, PL2-, PL3-, and K-schemas, plus the T-schema:

$$\Box\phi\rightarrow\phi \qquad \text{(T)}$$

T theorem:

$$\phi\rightarrow\Diamond\phi \qquad (\text{T}\Diamond)$$

Axiomatic system B:

- Rules: MP, NEC
- Axioms: the PL1-, PL2-, PL3-, K-, and T-schemas, plus the B-schema:

$$\Diamond\Box\phi\rightarrow\phi \qquad \text{(B)}$$

B theorem:

$$\phi\rightarrow\Box\Diamond\phi \qquad (\text{B}\Diamond)$$

Axiomatic system S4:

- Rules: MP, NEC
- Axioms: the PL1-, PL2-, PL3-, K-, and T-schemas, plus the S4-schema:

$$\Box\phi\rightarrow\Box\Box\phi \qquad (\text{S4})$$

S4 theorem:

$$\Diamond\Diamond\phi\rightarrow\Diamond\phi \qquad (\text{S4}\Diamond)$$

Axiomatic system S5:

- Rules: MP, NEC
- Axioms: the PL1-, PL2-, PL3-, K-, and T-schemas, plus the S5-schema:

$$\Diamond\Box\phi\rightarrow\Box\phi \qquad (\text{S5})$$

S5 theorem:

$$\Diamond\phi\rightarrow\Box\Diamond\phi \qquad (\text{S5}\Diamond)$$

Table 1: Some tautologies

$\phi \leftrightarrow \sim\sim\phi$	(double negation)
$(\phi \rightarrow \psi) \leftrightarrow (\sim\psi \rightarrow \sim\phi)$	(contraposition)
$\big((\phi \rightarrow \psi) \wedge (\psi \rightarrow \chi)\big) \rightarrow (\phi \rightarrow \chi)$	(syllogism)
$\big(\phi \rightarrow (\psi \rightarrow \chi)\big) \leftrightarrow \big((\phi \wedge \psi) \rightarrow \chi\big)$	(import/export)
$\big(\phi \rightarrow (\psi \rightarrow \chi)\big) \leftrightarrow \big(\psi \rightarrow (\phi \rightarrow \chi)\big)$	(permutation)
$\big((\phi \rightarrow \psi) \wedge (\phi \rightarrow \chi)\big) \leftrightarrow \big(\phi \rightarrow (\psi \wedge \chi)\big)$	(composition)
$\big((\phi \rightarrow \chi) \wedge (\psi \rightarrow \chi)\big) \leftrightarrow \big((\phi \vee \psi) \rightarrow \chi\big)$	(dilemma)
$\big((\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)\big) \leftrightarrow (\phi \leftrightarrow \psi)$	(biconditional)
$(\sim\phi \rightarrow \psi) \leftrightarrow (\phi \vee \psi)$	(disjunction)
$(\phi \rightarrow \sim\psi) \leftrightarrow \sim(\phi \wedge \psi)$	(negated conjunction)