

Modal Propositional Logic Theorems

K-Theorems

1. $\sim\Box\Diamond\Box P \leftrightarrow \Diamond\Box\Diamond\sim P$
2. $\sim\Diamond(P\vee Q) \rightarrow (\sim\Diamond P \wedge \sim\Diamond Q)$
3. $\Diamond(P\wedge Q) \rightarrow (\Diamond P \wedge \Diamond Q)$
4. $\Box(\sim P \rightarrow P) \leftrightarrow \Box P$
5. $\Box(P \rightarrow \sim P) \leftrightarrow \sim\Diamond P$
6. $\Box P \rightarrow \Box(Q \rightarrow P)$
7. $\Box\sim P \rightarrow \Box(P \rightarrow Q)$
8. $\sim\Diamond(Q\wedge R) \leftrightarrow \Box(Q \rightarrow \sim R)$
9. $[\Box(P \rightarrow Q) \wedge \Box(P \rightarrow \sim Q)] \rightarrow \Box(P \rightarrow \sim P)$
10. $(\Box P \wedge \Box Q) \rightarrow \Box(P \wedge Q)$
11. $\Box(P \leftrightarrow Q) \rightarrow (\Box P \leftrightarrow \Box Q)$
12. $(\Box P \vee \Box Q) \rightarrow \Box(P \vee Q)$
13. $[\Box(Q \rightarrow P) \wedge \Box(\sim Q \rightarrow P)] \leftrightarrow \Box P$
14. $[\Box(P \rightarrow Q) \wedge \Box(P \rightarrow \sim Q)] \rightarrow \sim\Diamond P$
15. $\Box(P \vee Q) \rightarrow (\Box P \vee \Diamond Q)$
16. $[\Box(P \rightarrow Q) \wedge \Box(Q \rightarrow R)] \rightarrow \Box(P \rightarrow R)$
17. $(\Box P \wedge \Box Q) \rightarrow \Box(P \leftrightarrow Q)$
18. $\Diamond(P \vee Q) \leftrightarrow (\Diamond P \vee \Diamond Q)$
19. $(\Diamond P \wedge \Box Q) \rightarrow \Diamond(P \wedge Q)$
20. $[\Box(P \rightarrow Q) \wedge \Diamond(P \wedge R)] \rightarrow \Diamond(Q \wedge R)$
21. $\Diamond(P \rightarrow Q) \leftrightarrow (\Box P \rightarrow \Diamond Q)$
22. $\Diamond P \rightarrow (\Box Q \rightarrow \Diamond Q)$
23. $\Diamond[P \rightarrow (Q \wedge R)] \rightarrow [(\Box P \rightarrow \Diamond Q) \wedge (\Box P \rightarrow \Diamond R)]$
24. $[\Box\Diamond P \wedge \Diamond\Box(P \rightarrow Q)] \rightarrow \Diamond\Diamond Q$

D-Theorems

- 25. $\Box\Box P \rightarrow \Box\Diamond P$
- 26. $\Box\Box P \rightarrow \Diamond\Diamond P$
- 27. $\Box P \rightarrow \Diamond(Q \rightarrow P)$
- 28. $\sim\Box(P \wedge \sim P)$
- 29. $[\Box P \wedge \Box(P \rightarrow Q)] \rightarrow \Diamond Q$
- 30. $\sim(\Box P \wedge \Box\sim P)$
- 31. $\Diamond\{[(P \rightarrow Q) \rightarrow P] \rightarrow P\}$
- 32. $\sim\Box[\Box(P \wedge Q) \wedge \Box(P \rightarrow \sim Q)]$
- 33. $(\Diamond\sim P \vee \Diamond\sim Q) \vee \Diamond(P \wedge Q)$

T-Theorems

- 34. $\Box P \rightarrow \Box\Diamond P$
- 35. $\Diamond\Box P \rightarrow \Diamond(P \vee Q)$
- 36. $[\Box P \wedge \Diamond\Box(P \rightarrow Q)] \rightarrow \Diamond Q$
- 37. $\Diamond(P \rightarrow \Box Q) \rightarrow (\Box P \rightarrow \Diamond Q)$

B-Theorems

- 38. $\Diamond\Box P \rightarrow \Box\Diamond P$
- 39. $\Diamond\Box P \leftrightarrow \Diamond\Box\Diamond\Box P$
- 40. $\sim\Diamond(\Diamond\Box\Diamond P \wedge \Box\sim P)$
- 41. $[\Box P \wedge \Box\Diamond\Box(P \rightarrow Q)] \rightarrow \Box Q$

S4-Theorems (43, 44, 47, 48, and 49 are also B-theorems)

- 42. $(\Diamond P \wedge \Box Q) \rightarrow \Diamond(P \wedge \Box Q)$
- 43. $\Box P \rightarrow \Box\Diamond\Box P$

44. $\Box\Diamond P \rightarrow \Box\Diamond\Box\Diamond P$
 45. $(\Box P \vee \Box Q) \rightarrow \Box(\Box P \vee \Box Q)$
 46. $\Box[\Box(P \leftrightarrow Q) \rightarrow R] \rightarrow \Box[\Box(P \leftrightarrow Q) \rightarrow \Box R]$
 47. $\Box\Diamond\Box\Diamond P \rightarrow \Box\Diamond P$
 48. $\Diamond\Box P \rightarrow \Diamond\Box\Diamond\Box P$
 49. $\Diamond\Box\Diamond\Box P \rightarrow \Diamond\Box P$

S5-Theorems

50. $\Diamond\Diamond\Diamond\Box P \leftrightarrow \Box P$
 51. $\Box\Diamond\Diamond\Box P \leftrightarrow \Box\Box P$
 52. $\Box(\sim P \vee \Box Q) \leftrightarrow (\Box\sim P \vee \Box Q)$
 53. $\Box(\sim P \vee \Diamond Q) \leftrightarrow (\sim\Diamond P \vee \Diamond Q)$
 54. $(\Box P \vee \Diamond Q) \leftrightarrow \Box(P \vee \Diamond Q)$
 55. $\Diamond(P \wedge \Diamond Q) \leftrightarrow (\Diamond P \wedge \Diamond Q)$
 56. $(\Diamond P \wedge \Box Q) \leftrightarrow \Diamond(P \wedge \Box Q)$
 57. $\Box(\Box P \rightarrow \Box Q) \vee \Box(\Box Q \rightarrow \Box P)$
 58. $\Box[\Box(\Diamond P \rightarrow Q) \leftrightarrow \Box(P \rightarrow \Box Q)]$