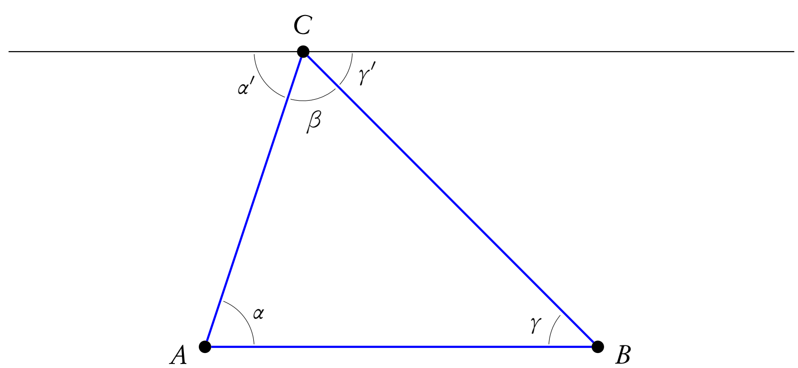
Kant

Ted Sider
Philosophy of Mathematics

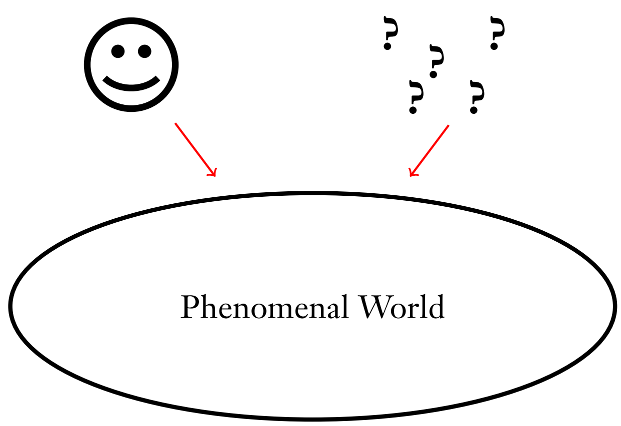
1. Kant on geometry

We use diagrams in proofs in geometry. Example: to prove that the angles of a triangle add up to 180°, we draw a triangle, $\triangle ABC$, and then a line through C that is parallel to $\overline{AB}$.



$\angle\alpha = \angle\alpha'$ since they are alternate interior angles of two parallel lines cut by a transversal. Similarly, $\angle\gamma = \angle\gamma'$. But $\alpha' + \beta + \gamma' = 180°$ since those three angles span a straight line. So $\alpha + \beta + \gamma = 180°$.

But how do we know that diagrams we construct or imagine accurately represent space? Kant's answer: the external world is constituted partly by how the world is in itself (the "noumenal world"), and partly by our minds:



And we can know a priori the rules governing what our minds contribute to this process. This, he said, lets us know a priori what space is like. In fact, he said, we know a priori that space is Euclidean.

2. Mathematics is synthetic

Kant calls a proposition *analytic* if "its predicate is contained in its subject". Example: 'All triangles have three angles'. Analytic sentences seem easy to know. So if mathematics is analytic, maybe there isn't such a problem about how we know it.

In a *synthetic* proposition, like 'Socrates is snubnosed', the predicate is not contained in the subject. So it's harder to know. You need to actually go and look at Socrates to tell whether he's snubnosed.

Kant thought that mathematical propositions like '12 is the sum of 7 and 5' and 'a triangle's angles add up to 180 degrees' are synthetic. Here is a "Kantian" argument (though not really Kant's own) for that conclusion: an analytic sentence never tells us that an object exist; it only tells us that *if* an object satisfies the subject, it also satisfies the predicate. (This is the moral he draws from the failure of Descartes's ontological argument for God's existence.). So insofar as mathematical statements assert that objects exist (referential use), they aren't analytic.

Another point against analyticity: some arguments *for* analyticity fail, such as Leibniz's. He gives these definitions:

$$2 = 1 + 1 \quad \text{(definition of '2)}$$
$$3 = 2 + 1 \quad \text{(definition of '3')}$$
$$4 = 3 + 1 \quad \text{(definition of '4')}$$

Then he argues from the definitions that $2 + 2 = 4$:

$$\begin{aligned} 2 + 2 &= 2 + 1 + 1 && \text{(using the definition of '2')} \\ &= 3 + 1 && \text{(using the definition of '3')} \\ &= 4 && \text{(using the definition of '4')} \end{aligned}$$

Problem: argument tacitly assumes that addition is associative, which has not been shown to be analytic.