Deductivism

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Deductivism

Mathematics investigates the logical consequences of axioms whose nonlogical expressions are uninterpreted

Thus mathematics establishes conditional statements:

> "if <axioms>, then <theorem>"

which would be true no matter how we interpreted the nonlogical expressions.

Advantages/features:

> No need to explain how we know the axioms
>
> No need to explain what mathematics is about
>
> Doesn't imply that proofs are arbitrary
>
> Meshes with/relies on abstract/formalized conception of mathematics

1. Deductivism and applied mathematics

If we can give a scientific interpretation to mathematical language under which the axioms are true, we would then know that the theorems are true.

This might work for geometry.

But suppose we want to use arithmetic for counting place-settings:

> How to interpret arithmetic predicates?
>
> How will the axioms come out true, given that there are only finitely many place-settings?

2. Deductivism and mathematical logic

Is knowledge of what follows from axioms really unproblematic?

The problem is sharpened if we "mathematize" logic, as Hilbert did—if we give mathematically rigorous definitions of logical concepts. For example:

Definition of proof

A *proof of conclusion* C *from premises* $P_1, \ldots, P_n$ is defined as a finite sequence of formulas, the last of which is C, in which each formula is either i) an axiom of logic, or ii) one of the premises $P_1, \ldots, P_n$, or iii) follows by a rule of inference from earlier formulas in the series.

We can similarly give mathematical definitions of *formula*, *axiom of logic*, and *rule of inference*. They're all a matter of the "shapes" of strings of symbols.

Then we define "following from":

Definition of following from

A formula C *follows from* formulas $P_1, \ldots, P_n$ if and only if there exists some proof of C from $P_1, \ldots, P_n$

Thus deductivists are committed to the *truth* of statements like these:

> There exists some proof—that is, a certain sequence of strings—of the string '$\forall x \forall y\, x + y = y + x$' from the strings that are the axioms of arithmetic

So at least one portion of mathematics can't be the mere investigation of the logical consequences of uninterpreted axioms, namely, the theory of strings.