Intro to Phil of Math

Ted Sider
Philosophy of Mathematics

1. Epistemological puzzles

Mathematics is a puzzle for epistemology (= philosophical study of knowledge) because, although it is clear *that* we have mathematical knowledge, it isn't clear *how* we have it.

1.1 Mathematics is a priori

A posteriori knowledge derived from the senses (e.g., science)

A priori knowledge not derived from the senses

Mathematics seems a priori. We learn about it by calculation, and by proof.

1.2 Puzzles about a priori knowledge

A posteriori knowledge is (comparatively) easy to understand. We know about things using our senses by causally interacting with them.

But a priori knowledge is puzzling! How can we know about something without causally interacting with it? If $2+4$ suddenly stopped being 6, we would never know the difference!

Relatedly: how do we know that mathematical *axioms* are really true?

2. Metaphysical puzzles

What are mathematical entities like? They seem not to be physical objects, but rather "abstract":

> We are told that the number zero was discovered in India, but it would be a mistake to go to India *now* to look for it—and not because it has subsequently been moved. You can't trip over the number three. The polynomial (x^2-3x+2) can be split into two factors, $(x-2)$ and $(x-1)$, but not by firing integers at it in a particle accelerator. The empty set has no gravitational field. And so on. (Donaldson, 2020, p. 709)

Are there really any such weird objects?

2.1 Arithmetic

Two kinds of uses of number words:

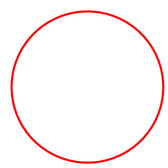
Adjectival "The US has fifty states", "Michael Jordan has six rings"

Referential "the number 6 is what you get when you add the number 4 to the number 2—i.e., $6 = 4 + 2$", "There are infinitely many prime numbers"

It is the referential use that is most perplexing. What is this object named by '3'? (We need to distinguish numerals from numbers. '3' and 'III' are distinct numerals, but name the same number.)

2.2 Geometry

What are, e.g., circles? Even a well-drawn "circle" isn't a true circle.



Another example: in his *Elements*, Euclid says that between any two points, there always exists (exactly) one line extending infinitely in both directions:

But we can't actually actually draw an infinitely long line.

Is geometry about marks on page? Parts of space? "Ideal" geometric objects?

References

Donaldson, Thomas (2020). "David Armstrong on the Metaphysics of Mathematics." *Dialectica* 74: 113–136.