LOGICISM: NEO-FREGEANISM

Ted Sider
Phil of Math

1. Neo-Fregeanism

Hume's Principle the number of the Fs = the number of the Gs if and only if F and G are equinumerous.

Frege's Basic Law V The extension of F = the extension of G if and only if: for any object x, x has F if and only if x has G

We now know that arithmetic can be derived solely from Hume's Principle (in second-order logic), without the need for Basic Law V, and that Hume's Principle—unlike Basic Law V—is (probably) not inconsistent. This has recently led some to adopt the following view:

Neo-Fregean view of arithmetic Hume's Principle is a definition of 'number of'. Since arithmetic logically follows from this definition, logicism for arithmetic is vindicated.

2. Is Hume's Principle really a definition?

Explicit definitions provide a synonym for the word being defined; e.g.:

> 'Tricycle' means 'vehicle with three wheels'.

Hume's principle isn't an explicit definition of 'the number of'. At best, it's an "implicit" definition: a stipulation that the meaning of 'the number of' is such as to make Hume's Principle true.

But implicit definitions face two pitfalls.

3. The problem of uniqueness

Pitfall one: they might fail to secure a *unique* meaning. E.g.:

> Implicit definition of 'Tiber Septim' using: 'Tiber Septim is a person'.

Hume's Principle doesn't secure a unique meaning for 'number of', since it doesn't tell us whether the number of fish in the ocean = Julius Caesar.

Possible reply: 'number of' is *vague*. This would fit with the abstract/structural conception of mathematics, since whether a number is Julius Caesar isn't a structural matter.

4. The problem of existence

Pitfall two: they might not secure *any* meaning. E.g.:

> Implicit definition of 'Frampt' using: 'Frampt gave me a million dollars'.

If there are only finitely many objects, then *no* relation between concepts and objects satisfies Hume's Principle. So if we don't *already* know that numbers exist, we don't know whether Hume's Principle is a successful implicit definition.

Possible reply: "ontological deflationism". The existence of (certain kinds of) objects is a trivial matter; we can know that they exist simply by reflecting on how we use language.

5. Bad company objection

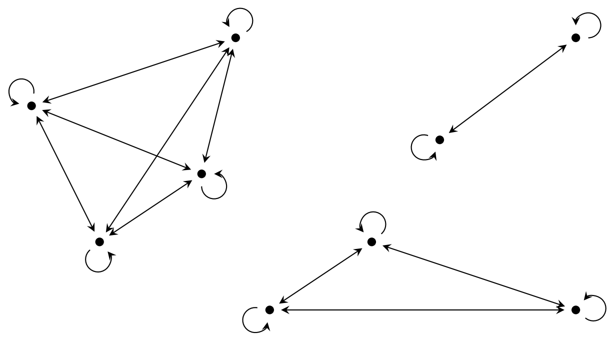
Hume's Principle is similar to inconsistent principles.

> The number of F = the number of G if and only if: F and G are equinumerous (Hume's Principle)

> The extension of F = the extension of G if and only if: for any object x, x has F if and only if x has G (Basic Law V)

The similarity is that they're both "definitions by *abstraction*".

In a definition by abstraction, you start with an equivalence relation (transitive, symmetric, reflexive):



You then introduce new entities corresponding to the nonoverlapping groups of mutually related things.

Example: begin with the equivalence relation *(full) sibling of*, and introduce "sib-squads" by abstraction:

> The sib-squad of person $x =$ the sib-squad of person y if and only if x is a sibling of y

Example: begin with the equivalence relation *is parallel to* over lines, and introduce "directions" by abstraction:

> The direction of line $l_1 =$ the direction of line l_2 if and only if l_1 is parallel to l_2

The bad company objection is that Hume's Principle is a definition by abstraction, and definitions by abstraction are sometimes illegitimate since they're sometimes inconsistent (Basic Law V).

Reply: consistent abstraction principles are legitimate.

Counter-reply: to know that an abstraction principle is legitimate, we would need to prove that it is consistent, which would require already knowing some mathematics.

Counter-reply: we can't accept all consistent abstraction principles, since some individually consistent abstraction principles are inconsistent with each other.

Example (from George Boolos). First define this equivalence relation: F and G *differ evenly* if and only if the number of objects falling under F but not G, or under G but not F, is even (and finite). Then introduce "parities" by abstraction:

Parity Principle The parity of F = the parity of G if and only if F and G differ evenly

The Parity Principle is consistent, as is Hume's Principle, but they are inconsistent with each other (Hume's Principle implies that there are infinitely many things, and the Parity Principle implies that there are only finitely many things).

6. Can the account be extended?

Can analysis, set theory, and other branches of mathematics be based on definitions by abstraction?