

STRUCTURALISM

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Structuralist slogan: “Only structure matters”

1. Set theoretic platonism and structuralism

The acceptability of defining mathematical objects as sets relies on the slogan, since what the definitions capture is structural features.

(Thus multiple set-theoretic definitions would be adequate. E.g. we could reverse \emptyset and $\{\emptyset\}$ in the sequence of natural numbers.)

(Thus perhaps ordinary talk of natural numbers is massively ambiguous.)

But sets themselves don't fit the slogan. We are talking about particular entities when we say that set A is a member of set B .

2. Ante rem structuralism

To more closely adhere to the slogan, ante rem structuralists posit new entities: *structures* and *positions in structures*.

(A baseball defense is a kind of structure, containing the positions of short-stop, center field, etc. The structure and the positions exist independently of being exemplified by particular baseball players.)

Ante rem structuralists say that mathematical objects *are* positions in structures. E.g., consider the *natural number structure*:

p_0 p_1 p_2 ...

This is the structure that is exemplified by any sequence of entities with the same structure as the natural numbers. The number 0 is the first position in this structure. That is, the number 0 is the entity p_0 .

Objection: there are other sequences with the very same structure, such as:

Julius Caesar p_1 p_2 ...

Given the slogan that “only structure matters”, why should only the first sequence count as the natural numbers?

3. An unsolvable problem?

Any view about the foundations of mathematics will put forward particular entities structured in some particular way to undergird mathematics; so it seems that no such view could comply with the slogan.

(At least, this is true if the *point* of foundations of mathematics is to put forward some particular view about what mathematical reality is like.)

(Even set-theoretic pluralists do this: they put forward one particular set-theoretic “pluriverse”, as opposed to other consistent ones.)

Possible solution: the slogan is about methodology, not metaphysics. Mathematics only *cares* about structure, but more might be *true* beyond structure.