Kant

Ted Sider
Philosophy of Mathematics

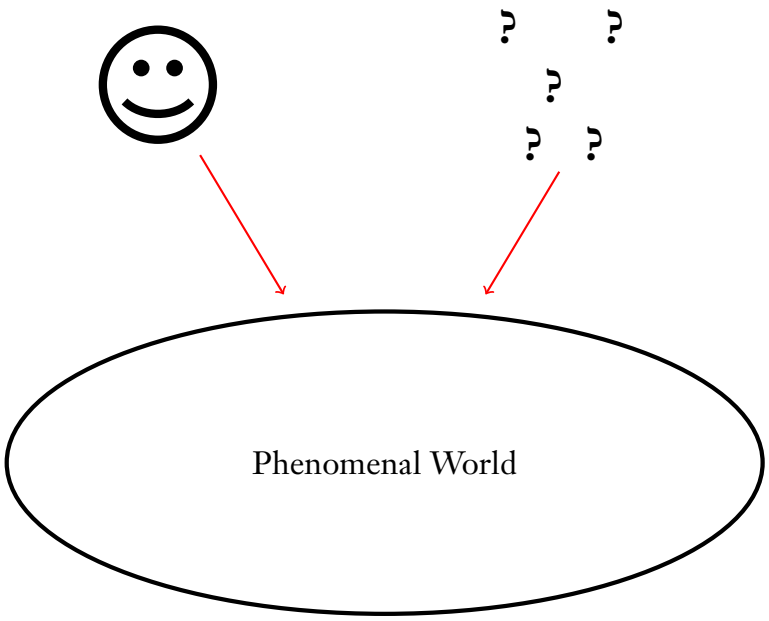
1. Reply to Hume

17th and 18th century debate over how to justify belief in the external world:

Descartes: we can be certain about how things *seem* to us from the inside; but how to build up to the external world?

Hume: we can't. (i) Knowledge of the external world requires knowledge of causation. (ii) Causal statements are matters of fact; and matters of fact can't be known a priori (Hume's empiricism). (iii) But causal statements can't be known a posteriori either, because we don't perceive causation itself and can't noncircularly argue that the future will resemble the past.

Kant: we can know facts about causation a priori, even though they're matters of fact (or rather, synthetic), because facts about causation, like all facts about the external world, are constituted partly by how the world is in itself (the "noumenal world"), and partly by our minds' operation; and we can know a priori the rules by which our mind operates.



2. Mathematics is synthetic

For Kant, a proposition is analytic if "its predicate is contained in its subject"; synthetic otherwise. Kant says that mathematics is synthetic.

Argument in Kantian terms (though not really Kant's own argument): an analytic sentence never tells us that an object exist; it only tells us that *if* an object satisfies the subject, it also satisfies the predicate. (This is the moral he draws from the failure of Descartes's ontological argument for God's existence.). So insofar as mathematical statements assert that objects exist (referential use), they aren't analytic.

Another point against analyticity: some natural arguments *for* analyticity fail, such as Leibniz's:

$$\begin{aligned} 2 &= 1+1 && \text{(definition of '2')} \\ 3 &= 2+1 && \text{(definition of '3')} \\ 4 &= 3+1 && \text{(definition of '4')} \end{aligned}$$

Argument from the definitions that $2+2=4$:

$$\begin{aligned} 2+2 &= 2+1+1 && \text{(using the definition of '2')} \\ &= 3+1 && \text{(using the definition of '3')} \\ &= 4 && \text{(using the definition of '4')} \end{aligned}$$

Problem: argument tacitly assumes that addition is associative, which has not been shown to be analytic.

3. Mathematics is nevertheless a priori

Because it's about the structure of the phenomenal world (both its spatial and temporal structure), which is partly constituted by rules of our mind, which we can discern a priori.