Intro to Phil of Math

Ted Sider
Philosophy of Mathematics

1. Epistemological puzzles

1.1 Mathematics is a priori

Mathematics seems a priori. We learn about it by calculation, and by proof.

1.2 Puzzles about a priority

A priori knowledge is puzzling. A posteriori knowledge, e.g. sensory knowledge, is easier to understand: we causally interact with our environment. But we don't causally interact with mathematical objects, since they are "abstract":

> We are told that the number zero was discovered in India, but it would be a mistake to go to India *now* to look for it—and not because it has subsequently been moved. You can't trip over the number three. The polynomial $(x^2 - 3x + 2)$ can be split into two factors, $(x - 2)$ and $(x - 1)$, but not by firing integers at it in a particle accelerator. The empty set has no gravitational field. And so on. (Donaldson, 2020, p. 709)

2. Metaphysical puzzles

What are mathematical entities like? We have no clear sense of their nature.

2.1 Arithmetic

Two kinds of uses of number words:

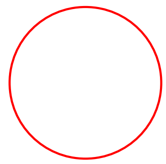
Adjectival "The US has fifty states", "Michael Jordan has six rings"

Referential "the number 6 is what you get when you add the number 4 to the number 2—i.e., $6 = 4 + 2$", "There are infinitely many prime numbers"

It is the referential use that metaphysically perplexing. What is this object named by '3'?

2.2 Geometry

What are, e.g., circles? Even a well-drawn "circle" isn't a true circle.



Another example: in his *Elements*, Euclid says that between any two points, there always exists (exactly) one line extending infinitely in both directions:

But we can't actually actually draw an infinitely long line.

Is geometry about marks on page? Parts of space? "Ideal" geometric objects?

3. Two complementary challenges

How difficult the epistemological challenge is can depend on how we answer the metaphysical challenge

References

Donaldson, Thomas (2020). "David Armstrong on the Metaphysics of Mathematics." *Dialectica* 74(4): 113–136.