Logicism: Neo-Fregeanism

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1. Neo-Fregeanism

Hume's Principle The number of the Fs = the number of the Gs if and only if F and G are equinumerous.

Frege's Basic Law V The extension of F = the extension of G if and only if: for any object x, x has F if and only if x has G

We now know that arithmetic can be derived solely from Hume's Principle (in second-order logic), without the need for Basic Law V, and that Hume's Principle—unlike Basic Law V—is (probably) not inconsistent. This has recently led some to adopt the following view:

Neo-Fregean view of arithmetic Hume's Principle is a definition of 'number of'. Since arithmetic logically follows from this definition, logicism for arithmetic is vindicated.

2. Is Hume's Principle really a definition?

Hume's Principle isn't an "explicit" definition, since it doesn't give a synonym for 'number of'.

According to some, there are also "implicit" definitions: a term is stipulated to have a meaning such that certain specified sentences come out true.

One view is that implicit definitions attempt to *select* the phrase's meaning:

> Consider all of the relations between concepts and objects in the entire world. You want to know which one I'm talking about, when I say 'number of'? I'll tell you: it's the one that satisfies Hume's Principle. That is, it is the relation, R, such that Hume's Principle comes out true if 'number of' is interpreted as meaning R.

Two lurking problems: an implicit definition might not single out a *unique* meaning for the word being defined; and the alleged meaning may not *exist*.

3. The problem of uniqueness

There may be many relations R satisfying Hume's Principle. This is the Caesar problem. Hume's Principle doesn't settle questions like whether the number of fish in the ocean = Julius Caesar.

NeoFregeans might say that the answers to such questions are *vague* (or "unsettled"). This would fit with the abstract/structural conception of mathematics, since those questions are about the nonstructural features of numbers.

4. The problem of existence

Some implicit definitions fail because they fail to select anything at all (e.g., "Frampt is the person who gave me a million dollars").

One might object that to *know* that Hume's Principle is a successful implicit definition, you'd need to know that there is at least one relation between concepts and objects satisfying it. But there is such a relation only if there exist infinitely many objects.

Possible response: ontological deflationism.

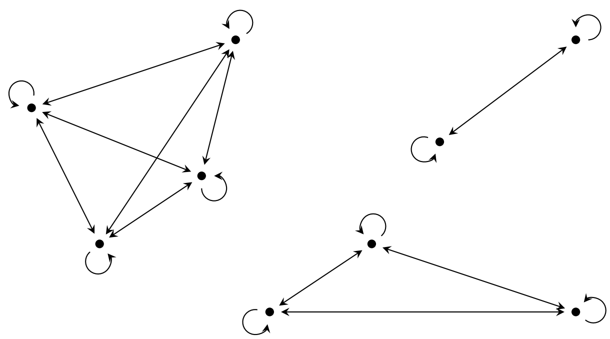
5. Bad company objection

Some definitions similar to Hume's Principle are inconsistent, e.g., Frege's Basic Law V:

> The extension of F = the extension of G if and only if: for any object x, x has F if and only if x has G (Basic Law V)

> The number of F = the number of G if and only if: F and G are equinumerous (Hume's Principle)

Each is a "definition by *abstraction*", in which we begin with an equivalence relation (relation that is reflexive, symmetric, and transitive), and introduce objects corresponding to the relation's equivalence classes:



Example: begin with the equivalence relation *(full) sibling of*, and introduce "sib-squads" by abstraction:

> The sib-squad of x = the sib-squad of y if and only if x is a sibling of y

Example: begin with the equivalence relation *is parallel to* over lines, and introduce "directions" by abstraction:

> The direction of line l_1 = the direction of line l_2 if and only if l_1 is parallel to l_2

General form:

> the BLAH of F = the BLAH of G if and only if EQUIVALENCE-RELATION holds between F and G

Objection: definitions by abstraction aren't always legitimate, because sometimes they are inconsistent (e.g., Basic Law V)

Reply: consistent abstraction principles are legitimate.

Counter-reply: to know that an abstraction principle is legitimate, we would need to prove that it is consistent, which would require already knowing some mathematics.

Counter-reply: we can't accept all consistent abstraction principles, since some individually consistent abstraction principles are inconsistent with each other. Boolos defined this equivalence relation: F and G *differ evenly* if and only if the number of objects falling under F but not G, or under G but not F, is even (and finite). Then introduce "parities" by abstraction:

Parity Principle The parity of F = the parity of G if and only if F and G differ evenly

The Parity Principle is consistent, as is Hume's Principle, but they are inconsistent with each other (Hume's Principle implies that there are infinitely many things, and the Parity Principle implies that there are only finitely many things).

6. Can the account be extended?

Can analysis, set theory, and other branches of mathematics be based on definitions by abstraction?