Structure in Physical Spaces

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Meta. Structure

What does it mean to say that physical space is curved, or that physical spacetime is Minkowskian?

1. Spatial structure: the mathematical conception

Structure is built into the definitions of mathematical entities.

Definition of group A group is a pair $\langle S, *\rangle$, where S is a set and $*$ is a binary operation on S such that $*$ is associative, and for some $e \in S$,

1. for each $a \in S$, $a * e = a$ and $e * a = a$
2. For each $a \in S$, there is some member of S, call it a^{-1}, such that $a * a^{-1} = e$ and $a^{-1} * a = e$

Definition of ring A ring is a triple $\langle S, *, \circ\rangle$, where S is a set, $*$ and $\circ$ are binary operations on S, $\langle S, *\rangle$ is a group, $*$ is commutative, $\circ$ is associative, and $\circ$ distributes over $*$

Definition of Tarski-space A Tarski space is a triple, $\langle S, B, \equiv\rangle$, where S is a set (the set of "points"), B is a three-place relation on S ("betweenness"), $\equiv$ is a four-place relation on S ("congruence"), and certain conditions including the following hold (for any $a, \ldots \in S$):[1]

1. if $Baba$ then $a = b$
2. if $ab \equiv cd$ and $ab \equiv ef$ then $cd \equiv ef$
3. there exists some $x \in S$ such that $Babx$ and $bx \equiv cd$

Whether a Tarski space is Euclidean depends on which conditions we lay down on B and $\equiv$.

[1] See Tarski and Givant (1999).

2. Structure in physical spaces: the problem

Physical spaces aren't given by definitions in the same way. There are relations over the points of space that satisfy Euclidean axioms, and relations that satisfy nonEuclidean axioms. Which relations are the *real*, or *spatial*, or the *physically significant* ones? Physicists talk about, e.g., in Minkowskian spacetime, there being no *distinguished* relation of simultaneity. What does this mean?

Reichenbachian conventionalism brings out these issues in a linguistic setting. Nonobservational terms (like 'physically between' and 'physically congruent'—'B_p' and '$\equiv_p$') need coordinative definitions. The following are two coordinative definitions of 'physically congruent'. If we adopt the first we'll describe space as being nonEuclidean; if we adopt certain definitions of the second form then we'll describe space as being Euclidean.

Definition 1 There are no universal forces; and $ab \equiv_p cd$ iff a measuring rod whose endpoints are a and b could be moved so that its endpoints are c and d.

Definition 2 There are such-and-such universal forces; and $ab \equiv_p cd$ iff a measuring rod whose endpoints are a and b could be moved so that one endpoint is c and an adjusted endpoint on the rod (or else the endpoint of an appropriate other rod appended to the first), depending on the strength of the universal forces, is d

3. Natural relations save the day

4. Structure in mathematics

What structure do the natural numbers (nonnegative integers) have?

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References

Tarski, Alfred and Steven Givant (1999). "Tarski's System of Geometry." *Bulletin of Symbolic Logic* 5: 175–214.