

# EQUIVALENCE

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Structuralism seminar

## 1. Equivalence

“Equivalent” theories represent the very same state of the world; any differences are merely conventional or notational.

## 2. Symmetry

*Symmetry* (of the laws): a one-one mapping from the statespace onto itself that maps allowed points to allowed points, and disallowed points to disallowed points.

Symmetry surely plays a central role in the epistemology of equivalence, but my interest is in the metaphysics of equivalence.

## 3. Examples: quantities, metric, ontology

*Kilo*: thinks that the kg scale for mass is “distinguished”.

*Grünbaum*: denies that space has a distinguished metric.

*Hirsch*: denies a distinguished concept of existence.

(Van Inwagen’s claim “there exist subatomic particles in a certain ‘chair-like’ arrangement *C*; there does not exist any further object (a chair) containing those particles as parts” and Lewis’s claim “there exist subatomic particles in arrangement *C*, and there also exists a further object containing those subatomic particles as parts” are equivalent.)

But what does this talk of “distinguished” concepts amount to?

## 4. Equivalence and fundamental concepts

**Fundamentality approach to equivalence** equivalent theories are those that say the same thing about the world at the fundamental level, about the fundamental concepts; “distinguished” concepts are fundamental ones

For example, Kilo might say that the mass-in-kilograms relation between massive objects and real numbers is a fundamental relation, and that mass-in-grams, mass-in-pounds, etc., are not.

What exactly does it mean to say that two statements “say the same thing about the world at the fundamental level” mean?

*Idea 1:* something grounds one iff it grounds the other

*Idea 2:* necessarily, something grounds one iff it grounds the other

*Idea 3:* they have the same “fundamental metaphysical analysis”

## 5. Difficult choices

Suppose we reject Kilo’s view, and say that *none* of the relations *mass-in-unit-U* is fundamental. What then are the fundamental properties or relations of mass? Perhaps:

- $x \succeq y$ :  $x$  is at least as massive as  $y$
- $Cxyz$ :  $x$  and  $y$ ’s combined masses equal  $z$ ’s

Given representation and uniqueness theorems, statements using different units of measure could have the same metaphysical analysis.

*Feature of this approach to equivalence:* in order to say that two theories (such as the g and kg theories) are equivalent, we must find a *third*, more fundamental, theory from whose point of view the first two theories are equally good ways of getting at the same facts.

What if there is no such third theory?

In the case of Hirsch’s claim that van Inwagen and Lewis’s theories are equivalent, it’s hard to see what the third theory could be. This is arguably not a problem for the fundamentality approach to equivalence, since Hirsch is arguably wrong.

But in other cases we want very much to claim equivalence despite the unavailability of a third theory:

- $\forall$  vs  $\exists$
- $\succeq$  vs  $\preceq$
- Parthood vs overlap vs fusion

Realism about concept-fundamentality apparently implies that it's a genuine question in each case which concept is fundamental. And the fundamentality approach to equivalence apparently implies that theories choosing different primitive notions from these lists are inequivalent.

## 6. “Quotienting out” conventional content “by hand”

A second approach to equivalence (rough statement): it's appropriate to say *that* theories are equivalent without saying *why* they are equivalent (in terms of third theories).

Example:

“A good theory can be formulated using the concept of  $\forall$ . But one can formulate an equivalent theory using the concept of  $\exists$  instead. Indeed, we can define a relation between theories that guarantees equivalence: differing solely by exchanges of formulas  $QvA$  and  $\sim Qv\sim A$ . True, we cannot provide a third, ‘more fundamental’ description of quantificational reality underlying this relation. But no such theory is needed; it's enough simply to say that theories standing in the relation are equivalent.”

I think that metaphysicians tend to assume something like the following:

“It's ok to construct models of some phenomenon, with artifacts. But there must also be some way of describing the phenomenon that in some sense does not have artifacts, some way of saying what is really going on. For example, modeling mass with real numbers is fine, but we need an underlying artifact-free description, such as the  $\succeq$  and  $C$  description, from which one can recover a specification of which numerical models are acceptable, and a specification of which features of these models are artifacts.”

Whereas this second approach rejects this assumption:

“There may be no way to say what is ‘really’ going on; maybe every good model has artifacts. It’s ok to just say: this model does a good job of representing the phenomenon, but certain features of the model are artifacts. Moreover, for any model, we can say which features of the model are genuinely representational and which are artifacts. There is no need to provide some privileged description that has no artifacts from which we can recover this information.”

**Quotienting** (Second approach, fuller statement) Given a set of theories with conventional differences, one can “quotient out” the conventional content and regard the best description as an equivalence class of theories. Moreover, one can do this “by hand”: the equivalence relation doesn’t have to be induced by some more fundamental theory, but rather can simply be stipulated.

## 7. Quotienting and structuralism

Quotienting is in some ways a good fit for structuralists. A structuralist who likes quotienting could avoid the problems discussed in previous lectures, and say merely that varying nodes while leaving the pattern intact results in an equivalent theory. No need for node-free descriptions of patterns, just an equivalence relation (defined by hand) on descriptions of patterns.

- Nomic essentialists could say merely that theories differing only by a permutation of scientific properties in the laws are equivalent.
- Structural realists and mathematical structuralists could say merely that theories differing solely by a permutation of individuals are equivalent.
- Also, manual quotienting allows “scooping out” arbitrary “aspects” of reality’s structure to which the laws are insensitive.

## 8. Stalnaker

Old problem for “actualism” about possible worlds: how to construct worlds containing entities that do not in fact exist? E.g. for a nonactual pair of duplicate dice being rolled, there should be two worlds in which they add up to 3:  $1/2$  and  $2/1$ . Stalnaker’s intriguing solution (*Mere Possibilities*):

- In Stalnaker’s Kripke models, the elements of the set  $W$  are called “points” rather than “worlds”, for reasons that will emerge.
- There are two points corresponding to the possibility of the dice summing to 3: one for  $2/1$ , the other for  $1/2$ .
- The particular entities that constitute the points are sometimes unimportant. (They could be bananas, or fish, or whatever; “it’s only a model”.) Thus the unavailability of intrinsically suitable representatives (e.g., haecceities) of the dice is not a problem; we can arbitrarily choose actual entities—bananas, say—to represent them.
- Since neither banana fixedly represents some nonactual die, neither of the pair of points represents anything different from the other, when taken individually. Each represents the possibility of a pair of nonactual dice summing to 3. The presence of one banana rather than the other is not representationally significant in these points, when taken individually.
- But the fact that the model contains *two* points rather than one *is* representationally significant in the context of the model as a whole: it represents the modal fact that had two such dice existed, there would have been two possibilities in which they sum to 3.
- Because of the arbitrariness of various choices (bananas), many different Stalnakerian models are representationally equivalent.
- Stalnaker thus defines an equivalence relation over his models, thus quotienting out the artifactual content.
- And he does this “by hand”. He doesn’t give any further account of worlds or modality that induces the equivalence relation.
- His attitude is: modal reality is such as to be well-modeled in this way, and there’s no need to give any further, artifact-free account of modality reality that shows *why* this is the case.

## 9. Against Quotienting

- Quotienting not normally pursued except when there's no choice. (We seek representation theorems for quantities, we seek coordinate-free formulations of geometric theories, and the default is (usually) that ontology anyway isn't conventional.)
- Manual quotienting is intuitively unsatisfying. (The equivalence relation a strange place for an unexplained explainer. Imagine Leibniz had said to Clarke merely that theories of space are equivalent when they differ only by some combination of translations and rotations.)
- If quotienters acknowledge in *some* cases that it's better to "explain" relations of equivalence in terms of a deeper theory, they concede that a theory in need of no quotienting would indeed be superior.

## 10. Defending the Fundamentality Approach

**Progress can be unexpected** (Can't be sure in advance that no reasons could ever be given.)

**Hard choices are hard to avoid** Hard to avoid acknowledging the existence of *some* choices like the difficult ones—grue/bleen and the like—and hard to acknowledge only them.

**There can be more than one** The question "is it  $\exists$  or  $\forall$  that is fundamental?" leaves out a third possibility: maybe they are *both* fundamental. (Parsimony not the only relevant consideration; avoidance of arbitrariness counts too.)

**Why think we can know everything?** The availability of the answer "both" mitigates the concern that there be no metaphysical asymmetry between  $\exists$  and  $\forall$ , but not the concern that the question is unanswerable.

Reply: yes, realism about fundamentality leads to some (apparently) unanswerable questions. That can be, but is not always, a sign that a concept fails to be in good standing. The concept of fundamentality arguably *is* in good standing.